

UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2006

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS
3. SHOW ALL THE RELEVANT WORKING

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow \infty} \frac{x+1}{x-1}$ [4 marks]

(ii) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$ [4 marks]

(b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions.

(i) $f(x) = \sqrt{2x}$ [6 marks]

(ii) $f(x) = \frac{1}{x}$ [6 marks]

QUESTION 2

2. Find the derivatives of the following functions

(a) $y = x^2 \ln x^2$ [5 marks]

(b) $y = x^{x^2}$ [5 marks]

(c) $y = \cos^2(x^2 + x + 1)$ [5 marks]

(d) $y = \ln \left(\frac{x^3 e^{x^3}}{\sqrt{x^2 + 1}} \right)$ [5 marks]

QUESTION 3

3. Find the following integrals

(a) $\int \left(3x^2 + e^{2x} + \sin 2x + \frac{3}{x} + \frac{1}{x^3} \right) dx$ [5 marks]

(b) $\int x^2 \sin x \, dx$ [5 marks]

(c) $\int \frac{2x - 1}{x^2 - 3x + 2} dx$ [5 marks]

(d) $\int x^2(x^3 + 1)^5 dx$ [5 marks]

QUESTION 4

4. (a) The marginal cost of producing x items of a product is given by $C'(x) = 0.04x$.

(i) Given that the fixed cost is E 20, find the total cost function $C(x)$. [3 marks]

(ii) Find the cost of producing 100 of these items. [2 marks]

(iii) Find the total change in cost if the number of items produced is changed from 100 to 200. [3 marks]

(b) A company manufactures and sells x computers per week. If the weekly cost and price-demand functions are given by

$$C(x) = 7500 + \frac{5x}{2} \quad \text{and} \quad p = \frac{21}{2} - \frac{x}{1000}$$

Find the following, for each week:

(i) the maximum revenue [6 marks]

(ii) the maximum profit [6 marks]

QUESTION 5

5. (a) Given the function $f(x) = x^3 - 6x^2 + 9x + 1$, find
- (i) the y -intercept [1 mark]
 - (ii) relative extrema [4 marks]
 - (iii) intervals of increase and decrease [3 marks]
 - (iv) intervals of concavity [4 mark]
 - (v) inflection points [2 mark]
- (b) Use all the information obtained in (a) to sketch a graph of the function. [6 marks]

QUESTION 6

6. Given the demand function $p = D(x) = 25 - 0.001x^2$ and the supply function $p = S(x) = 5 + 0.1x$, find
- (a) the equilibrium price [6 marks]
 - (b) the consumer's surplus [7 marks]
 - (c) the producer's surplus [7 marks]

QUESTION 7

7. (a) Find the area of the region bounded by the curves $y = x^2$ and $y = 5x - 6$ [8 marks]
- (b) Find the equation of the curve that passes through $x = -1$ if its slope is $y' = 2x + 5$ for any x . [6 marks]
- (c) If the marginal cost of producing x units is given by $C'(x) = 0.3x^2 + 0.2x + 1$ and the fixed cost is E500, find the total cost function $C(x)$. [6 marks]