

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2006

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS
3. SHOW ALL THE RELEVANT WORKING

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow 3} \frac{x+1}{x-1}$ [4 marks]

(ii) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$ [4 marks]

(b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions.

(i) $f(x) = x^2$ [6 marks]

(ii) $f(x) = \frac{1}{x}$ [6 marks]

QUESTION 2

2. Find the derivatives of the following functions

(a) $y = \ln x^2$ [5 marks]

(b) $y = x^x$ [5 marks]

(c) $y = \sin(x^2 + x + 1)$ [5 marks]

(d) $y = x^2 e^{x^2}$ [5 marks]

QUESTION 3

3. Find the following integrals

(a) $\int \left(3x^2 + \frac{3}{x} + \cos 2x \right) dx$ [5 marks]

(b) $\int x \cos x dx$ [5 marks]

(c) $\int \frac{2x - 1}{x^2 - 3x + 2} dx$ [5 marks]

(d) $\int 2x(x^2 + 1)^4 dx$ [5 marks]

QUESTION 4

4. (a) The marginal cost of producing x items of a product is given by $C'(x) = 0.04x$.

(i) Given that the fixed cost is E 20, find the total cost function $C(x)$. [3 marks]

(ii) Find the cost of producing 100 of these items. [2 marks]

(iii) Find the total change in cost if the number of items produced is changed from 100 to 200. [3 marks]

(b) A company manufactures and sells x computers per week. If the weekly cost and price-demand functions are given by

$$C(x) = 7500 + \frac{5x}{2} \quad \text{and} \quad p = \frac{21}{2} - \frac{x}{1000}$$

Find the following, for each week:

(i) the maximum revenue [6 marks]

(ii) the maximum profit [6 marks]

QUESTION 5

5. (a) A company manufactures and sells x radios per week. If the weekly cost and price-demand functions are given by

$$C(x) = 5000 + 2x, \quad p = 10 - 0.001x$$

Find the following, for each week.

- (i) the revenue function. [4 marks]
- (ii) the maximum revenue. [6 marks]
- (iii) the maximum profit. [6 marks]
- (iv) the price that will yield maximum profit. [4 marks]

QUESTION 6

6. Given the demand function $p = D(x) = 25 - 0.001x^2$ and the supply function $p = S(x) = 5 + 0.1x$, find

- (a) the equilibrium price [6 marks]
- (b) the consumer's surplus [7 marks]
- (c) the producer's surplus [7 marks]

QUESTION 7

7. (a) Find the area of the region bounded by the parabola $y = -x^2 - 6x$ and the line $y = 0$ [8 marks]
- (b) Find the equation of the curve that passes through (2,5) if its slope is given by $\frac{dy}{dx} = 2x$ at any point x . [6 marks]
- (c) If the marginal cost of producing x units is given by

$$C'(x) = 0.3x^2 + 2x$$

and the fixed cost is E2000, find the cost function $C(x)$ [6 marks]