UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2006

Dip.Comm. II/IDE-Dip.Comm. III

TITLE OF PAPER: Quantitative Techniques

COURSE NUMBER: MS202/IDE-MS202

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. This paper consists of SEVEN questions on FOUR pages. A formula sheet is provided.
2. Answer ANY FIVE questions.

SPECIAL REQUIREMENTS: NONE.

THIS EXAMINATION PAPER MUST NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

1. (a) The demand functions for two products are

$$p = 12 - x, \qquad q = 34 - y$$

where p and q are the respective prices (in thousands of Emalangeni) and x and y are the respective amounts (in thousands of units) of each product sold. If the joint cost function is given by

$$C(x, y) = x^2 + 2xy + 3y^2,$$

determine the quantities and prices that maximize profit (Verify they give maximum profit). What is the maximum profit?

[12 marks]

(b) An individual owes R20,000 due in 18 months at a rate of 12% simple interest. He decides to pay R10,000 after 6 months, R5,000 after 12 months, and the rest at due date. Determine the final payment, and the total interest paid.

[8 marks]

2. (a) Suppose the 4^{th} and 6^{th} terms in an arithmetic progression are 4 and 4.8 respectively. Find the first term.

[5 marks]

(b) The management of a factory wants to set up a fund to provide E40,000 for the replenishment of a machine at the end of five years. If equal deposits are made at the end of each period of SIX months in a fund earning 4% converted semi-annually, find the size of each deposit.

[5 marks]

(c) Maximize the profit function $P(x, y) = 5x^2 + 2xy + 3y^2 + 80$ subject to $y = -x + 24$ using the method of Lagrange multipliers.

[10 marks]

3. (a) Use Cramer's rule to solve

$$\begin{aligned} 2x - z &= 1 \\ 2x + 4y - z &= 1 \\ x - 8y - 3z &= -2 \end{aligned}$$

[10 marks]

(b) For the function, $z = 3x^3 - 5y^2 - 225x + 70y + 23$, determine all local extrema and classify them accordingly. **[10 marks]**

4. (a) A company that manufactures jackets requires 50,000 zippers a month. The ordering cost is E5000 and inventory holding costs are estimated to be 5^{c} per unit per month. Find the most economical order quantity associated with this business. **[5 marks]**

(b) A factory manufactures two products each requiring the use of three machines. Machine A can be used at most 70 hours, B at most 40 hours, and Machine C at most 90 hours. Each unit of the first product requires 2 hours on machine A, 1 hour on machine B, and 1 hour on machine C; each unit of the second product requires 1 hour each on machines A and B and 3 hours on machine C. The profit is E40 per unit for the first product and E60 per unit for the second product. Formulate an L.P.P. for determining the number of units of each product that should be manufactured to maximize profit. (Do not solve) **[5 marks]**

(c) A large railroad finds that it must steam clean its cars once a year. It is considering two alternatives for its steam cleaning operation. Under alternative 1, the railroad would operate two steam cleaning booths, operating in parallel, at a total annual cost of \$50 000. The service time distribution under this alternative is exponential with mean of 5 hours. Under alternative 2, the railroad would operate one large steam cleaning booth at a total cost of \$100 000. However, the service time distribution under this alternative would be exponential with a mean of 6 hours. Under both alternatives, the railroad cars arrive according to a Poisson input process with an arrival rate of one car every 8 hours. The cost of an idle hour is thought to be \$10 per hour. Assume that the steam cleaning booths operate 8 hours per day for 250 days a year. Which alternative should the railroad choose?

[10 marks]

5. Suppose that for a healthy diet an adult has a daily requirement of at least 10 units of energy and 15 units of protein and the following foods available:

	Bread	*Cheese*	*Meat*
Price per unit	2	4	5
Energy per unit	1	2	1
Protein per unit	1	3	6

(a) Set up the linear-programming problem for minimizing the total cost of satisfying the daily requirements.

[5 marks]

(b) Form the dual of the minimization problem and obtain the optimal solution.

[15 marks]

6. Dairy Maid company manufacturers its products in 4 factories and distributes to warehouses in 3 locations. The transport costs (in cents per carton of bottles and origin/destination requirements are given in the tableau below:

		Warehouse			
		1	2	3	Supply
	1	40	70	50	30
Factory	2	20	30	20	50
	3	30	60	60	30
Demand		50	20	40	

(a) Use the above information to construct a transportation tableau.

(b) Obtain an initial feasible solution using the north-west corner rule. Hence find the optimum shipment strategy and the associated cost.

[20 marks]

7. (a) Assume that radial tires were introduced on the market by three companies at the same time. When the tires were introduced, each firm had an equal share of the market, but during the first year, the following changes in the market share took palce:

Company A retained 80% of its customers; it lost 5% to B and 15% to C.

Company B retained 90% of its customers; it lost 10% to A and none to C.

Company C retained 60% of its customers; it lost 20% to A and 20% to B.

Construct the transition probability matrix and hence predict what the market shares will be after 2 years.

[12 marks]

(b) Let D, S and P denote demand, supply and price of each commodity in a two-commodity market (commodity 1 and 2) with linear model

$$\begin{aligned} D_1 &= 20 - 2P_1 - P_2 \\ S_1 &= 4P_1 - P_2 + 2 \\ D_2 &= 8 + 5P_1 - 2P_2 \\ S_2 &= 3P_2 - 2 \end{aligned}$$

Determine the equilibrium solution.

[8 marks]

********** END OF EXAMINATION **********

Note: A set of useful formulae appears on the next page.

Some useful formulae.

Hire Purchase Instalment: $p = \dfrac{B\left(1+\frac{n}{m}i\right)}{n+\frac{n(n-1)}{2m}i}$

Growing Investment: $S = \left(P + \dfrac{p}{i}\right)(1+i)^n - \dfrac{p}{i}$

Future Value of an Annuity: $S = p \cdot \dfrac{(1+i)^n - 1}{i}$

Present Value of an Annuity: $P = p \cdot \dfrac{1-(1+i)^{-n}}{i}$

For an EOQ model without shortages: $Q_0 = \sqrt{\frac{2dK}{h}}$

For a queueing model with one service channel and arrival and service rates λ, μ respectively:

$\rho = \frac{\lambda}{\mu}$; $p_0 = 1 - \frac{\lambda}{\mu}$ and $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$

$L_s = \frac{\lambda}{\mu-\lambda}$ $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$ $W_s = \frac{L_s}{\lambda}$ $W_q = \frac{L_q}{\lambda}$

(END)