

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2007

MS012 Year. I

TITLE OF PAPER: ELEMENTARY QUANTITATIVE METHODS

COURSE NUMBER: MS012

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. This paper consists of SEVEN questions on FOUR pages.
2. Answer any FIVE questions.
3. Calculators may be used.

SPECIAL REQUIREMENTS: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED
UNTIL PERMISSION HAS BEEN GRANTED BY THE
INVIGILATOR.

Question 1

- (a) What type of roots does the equation $2x^2 = px + p^2$ have? [4 marks]
- (b) The equation $x^2 - 2x + 1 = p(x - 3)$ has equal roots. Find the possible values of p . [6 marks]
- (c) Solve the equation $2x^3 - 3x^2 - 8x + 12 = 0$ [10 marks]

Question 2

- (a) Evaluate the following limits

(i)

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

[5 marks]

(ii)

$$\lim_{x \rightarrow \infty} \frac{x + 3}{x + 5}$$

[5 marks]

- (b) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x) = \sqrt{2x + 3}$$

[10 marks]

Question 3

(a) Find $f'(x)$ for each of the following functions

(i) $f(x) = x^5 + 2x^3 + x + 1$

[3 marks]

(ii) $f(x) = (x^2 + 5x)^7$

[4 marks]

(iii) $f(x) = e^{2x} \sin x + \cos 3x$

[5 marks]

(b) If $y = \frac{A}{x} + Bx$, where A and B are constants, show that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$$

[8 marks]

Question 4

(a) Evaluate the following integrals

(i) $\int (3x^4 + 5x^3 + x + 1) dx$

[3 marks]

(ii) $\int \sqrt{x+1} dx$

[5 marks]

(iii) $\int \cos(3x+2) dx$

[5 marks]

(b) Find the area enclosed by the curve $y = -x^2 + 9$ and the x -axis.

[7 marks]

Question 5

(a) For the function $f(x) = x^2 - 4x + 3$, find stationery points, intervals of decrease and increase.

[8 marks]

(b) Make a rough sketch of the curve by considering

$$f(x) = x^3 - 6x^2 + 9x + 1.$$

x and y intercepts, turning points, intervals of decrease and increase.

[12 marks]

Question 6

(a) A piece wire 48cm long is divided into two parts. One part is formed into the shape of a circle of radius r cm while the other part is formed into a square of side x cm.

(i) Show that $r = \frac{24 - 2x}{\pi}$

[4 marks]

(ii) Find an expression in terms of x for the total area A of the two shapes and hence calculate the value of x for which A is a minimum.

[6 marks]

(b) The cost of making x articles per day is $E(\frac{1}{2}x^2 + 50x + 50)$ and the selling price of each one is $E(80 - \frac{1}{4}x)$. Find

(i) the daily profit in terms of x

[5 marks]

(ii) the value of x to give the maximum profit.

[5 marks]

Question 7

(a) The first three terms of an AP are x , $2x + 1$, and $5x - 1$. Find the value of x and the sum of the first 10 terms.

[8 marks]

(b) Show that $x + 1$, $x + 3$ and $x + 5$ cannot be three consecutive terms of a GP , whatever the value of x .

[6 marks]

(c) Solve the equation $5^{x^2+1} = 6$.

[6 marks]

***** END OF EXAMINATION *****