University of Swaziland

Final Examination, December 2006

DCom I, Bass I, BEd com I

Title of Paper : Introductory Mathematics for Business
Course Number : MS101/IDE-MS101
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Expand $\left(2x - 1/x\right)^5$ and simplify term by term. [5 marks]

(b) Consider the sequence of numbers $4, -8, 16, \cdots$.

(i) Find the 20th term of the sequence. [3 marks]

(ii) Find the sum of the first 10 terms. [3 marks]

(c) Use synthetic division to find the quotient and remainder when $-x^3 + 8x^2 + 63$ is divided by $x + 2$. [4 marks]

(d) Express

$$4\log\sqrt{2} - \log(x^2+1) + 4\log(x+1)$$

as a single logarithm with coefficient 1. [5 marks]

Question 2

(a) Given the matrices

$$A = \begin{pmatrix} -1 & 2 & 3 \end{pmatrix},\ B = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix},\ C = \begin{pmatrix} 6 & -2 \\ -1 & 4 \end{pmatrix},\ D = \begin{pmatrix} 1 & -2 & 1 \\ 4 & 0 & 2 \end{pmatrix}.$$

Perform the following operations where possible. If an operation is impossible, clearly state so and give the reason why. [9 marks]

$$A + 2B,\ 4B - 2C^T,\ CD,\ DB,\ AD^T,\ DA.$$

(b) Consider the complex numbers $z_1 = 1+2i$ and $z_2 = 3-4i$. Compute

$$3z_1 - z_2,\ |z_2|,\ z_1 z_2,\ \tfrac{1}{z_1},$$

expressing your complex answers in the form $a + ib$. [7 marks]

(c) Solve for x if $27^{2x} = 9^{x-8}$. [4 marks]

Question 3

(a) Find the quotient and remainder when $P(x) = 2x^5 + x^4 - x^2 + 2x - 6$ is divided by $x^2 - 1$. [8 marks]

(b) Prove by mathematical induction [8 marks]

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \tfrac{1}{4}n^2(n+1)^2.$$

(c) If $\sin\theta = \frac{4}{5}$ and θ is in the second quandrant, find the exact values of $\cos\theta$ and $\tan\theta$. [4 marks]

Question 4

(a) Assuming the inflation rate is 4.5% compounded continuously, how long will it take for prices to double? [5 marks]

(b) Determine whether $x + 2a$ is a factor of $P(x) = 3x^4 + 4ax^3 - 2a^3x - 20a^4$. [5 marks]

(c) Find all fourth roots of -16, leaving your answer in the form $a + ib$. [10 marks]

Question 5

(a) Prove by mathematical induction [5 marks]

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 1.$$

(b) By regarding $1+2+2^2+2^3+\cdots+2^{n-1}$ as a Geometric Progression, state the common ratio. Hence show that the sum of these terms is given by $S_n = 2^n - 1$. [3 marks]

(c) Use Cramer's rule to solve the system [12 marks]

$$\begin{aligned} 2x - 3y - 3z &= 9 \\ 3y + 2z &= 0 \\ 3x - 4y &= 1. \end{aligned}$$

Question 6

(a) Find the centre and radius of the circle $x^2 + y^2 - 6x + 8y - 11 = 0$. [7 marks]

(b) Find all solutions of $\cos 2x + \cos x + 1 = 0$ in the range $0 \leq x < 360^0$. [7 marks]

(c) Solve $z^2 - 2iz - 5 = 0$. [6 marks]

Question 7

(a) Solve for x

(i) $\log_2(x^2-1) = 3 + \log_2(x-2)$ [5 marks]

(ii) $3^{2x+1} = 5^{x-1}$. [5 marks]

(b) Find the constant term in the binomial expansion of [5 marks]

$$\left(x^3 + \frac{1}{2x^2}\right)^{20}.$$

(c) Find the first 5 terms of the binomial expansion of $\sqrt{1-2x^2}$. [5 marks]

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