University of Swaziland

Final Examination, May 2007

DCom I, Bass I, BEd Com I

Title of Paper : Calculus for Business Studies

Course Number : MS102/IDE-MS102

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.

2. Each question is worth 20%.

3. Answer ANY FIVE questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find y' and simplify, given

    (i) $y = (1 + 2x) - (1 + 2x)\ln(1 + 2x)$ [6]

    (ii) $y = \ln\left(\dfrac{e^x}{1 - e^x}\right)$ [6]

(b) Integrate

$$\int x^2 \sin 2x \, \mathrm{d}x. \qquad [8]$$

Question 2

(a) Find the derivative of $f(x) = \left(1 - 2x^2\right)^3$.   [6]

(b) Evaluate

$$\int_1^4 \left(\frac{2}{x} + \frac{1}{2\sqrt{x}} - e^{-2x}\right) \mathrm{d}x. \qquad [6]$$

(c) A producer can sell x items per week at a price $p = 300 - 0.02x$, and it costs $C(x) = 40x + 50000$ to produce x units. Find the most profitable number of units to produce, and the weekly profit at this level of production.   [8]

Question 3

(a) Use the definition of the derivative to find $f'(x)$ for

$$f(x) = \frac{1}{1 - 2x}. \qquad [10]$$

(b) Consider the market characterised by the demand and supply functions

$$\begin{aligned} D(x) &= 0.1x^2 + 2x + 20 \\ S(x) &= 40 - x - 0.1x^2, \end{aligned}$$

respectively.

(i) Find the price p^* and the number of units x^* at market equilibrium. [5]

(ii) Work out the consumer's surplus. [5]

Question 4

(a) Differentiate and simplify each of the following functions

(i) $f(\theta) = \theta \sin \theta + \cos \theta$ [5]

(ii) $R(\varphi) = (\varphi^2 - 1)e^{\varphi^2}$ [5]

(b) Find

$$\int \frac{(2x+1)dx}{x^3 + x^2 + x + 1}.$$ [10]

Question 5

(a) Evaluate

(i) $\displaystyle\lim_{x \to \infty} \left(\frac{1 - 3x^2}{2x^2 - 101x + 50000} \right)$ [5]

(ii) $\displaystyle\int_{-1}^{0} (2 - x^2)^3 dx$ [6]

(b) Consider the function $f(x) = x^3 - 12x + 2$. Find and classify stationary and inflexion points, intervals where $f(x)$ is increasing/decreasing, and intervals where $f(x)$ is concave up/down. Hence sketch the graph of $f(x)$. [9]

Question 6

(a) Find y''' for the function $y = (x^2 - 1) \ln x$. [5]

(b) Show that

$$\int_0^{\sqrt{2}} \frac{x\,dx}{2 + x^2} = \ln \sqrt{2}.$$ [7]

(c) Find the exact value of the area for the region bounded between the curves $y = x^2$ and $y = 16 - x^2$. [8]

Question 7

(a) Find the derivative of $F(t) = \dfrac{t^2 - 1}{t^2 + 1}$ and simplify. [7]

(b) A company is characterised by the following marginal revenue and marginal cost functions

$$
\begin{aligned}
R'(x) &= \frac{240}{2 + x}, \\
C'(x) &= \frac{1}{8}x + 9,
\end{aligned}
$$

where x denotes the number of units sold. You are told that the company's sales manager has proposed that, to boost profitability, the company should increase sales from $x = 50$ presently to $x = 60$.

(i) Calculate the total change in profit if sales increase from 50 to 60. [9]

(ii) Based on your answer in (i), advise management whether implementing the sales manager's proposal will benefit the company. [4]

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