UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2007

BASS. I, D. COMM I

TITLE OF PAPER: CALCULUS FOR BUSINESS & SOCIAL SCI.

COURSE NUMBER: MS102/ IDE-MS102

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: 1. This paper consists of SEVEN
                        questions on FOUR pages.

                     2. Answer any FIVE questions.

                     3. Calculators may be used.

SPECIAL REQUIREMENTS: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED
UNTIL PERMISSION HAS BEEN GRANTED BY THE
INVIGILATOR.

Question 1

(a) Find the following limits

  (i) $\lim\limits_{x \to 5} \dfrac{x - 5}{x^2 - 25}$

  (ii) $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

[6 marks]

(b) Find the integral

$$\int \sin^3 x \cos^5 x \, dx$$

[6 marks]

(c) Use integration by partial fractions to evaluate

$$\int \frac{x - 2}{x^2(x - 1)} dx$$

[8 marks]

Question 2

(a) Find two positive numbers whose sum is 20 and whose product is as large as possible

[5 marks]

(b) Find the equation for the tangent to the curve at the point $(2, 3)$

[4 marks]

(c) Find the derivative of y with respect to x for the following

$$y = x^x$$

[5 marks]

(d) Fin the 1st and 2nd derivatives of the following

$$f(x) = x^2 \cos x$$

Question 3

(a) Find the area lying above the $x-$axis and under the parabola

$$y = 4x - x^2$$

[6 marks]

(b) Find the level of production that will minimize the average cost if the total cost of producing x units is given by

$$C(x) = 10000 + 5x + \frac{x^2}{9}$$

[8 marks]

(c) Determining the value of k given that the tangent to the curve

$$y = x^2 + k + 1$$

at the point $(0, 1)$ is parallel to the line $y - 2x = 4$.

[6 marks]

Question 4

(a) Find the value of the constant b for which the function

$$f(x) = \left\{ \begin{array}{ll} x^2 & if \quad x < 2 \\ bx & if \quad x \geq 2 \end{array} \right.$$

is continuous at $x = 2$

[6 marks]

(b) Evaluate that definite integral

$$\int_3^4 x(x^2 + 3)^8 dx$$

[7 marks]

(c) Use integration by parts to evaluate

$$\int x^2 \sin x dx$$

[7 marks]

Question 5

(a) A furniture shop's annual profit is stated a function of the number of salesmen x employed. The profit function is

$$p(x) = -12.5x^2 + 1375x - 1500$$

(i) Determine the number of salesmen which maximize annual profit
(ii) What is the maximum profit expected to equal?

[10 marks]

(b) Evaluate the following integrals
(i) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$
(ii) $\int \frac{x\,dx}{4 + x^2}$

[10 marks]

Question 6

(a) Use the definition to find the derivative $f'(x)$ of the following function

$$f(x) = 4 - 3x + 3x^2$$

[8 marks]

(b) Consider the function

$$f(x) = 3x^4 - 8x^2 - 12x^2 + 12x - 7$$

(i) Determine the intervals over which $f(x)$ is increasing or decreasing.
(ii) Find the relative extrema of $f(x)$.

[12 marks]

Question 7

(a) The total cost of producing q units of a certain product is

$$C(q) = 200000 + 5000q + 0.1q^2$$

(i) Determine how many units q should be produced in order to minimize the total cost.

[5 marks]

(ii) What is the total cost of production at this level of output

[5 marks]

(b) Calculate the area bounded by the curves
$y = x^2 - x$ and $y = 1 - x^2$

[10 marks]

******** *END OF EXAMINATION* ********