

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007

Dip.Comm II, IDE-Dip.Comm III

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF  
SEVEN QUESTIONS.  
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL  
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

### QUESTION 1

1. (a) Use the method of Lagrange multipliers to optimize

$$z = 80x - 2x^2 - xy - 3y^2 + 100y$$

subject to  $x + y = 12$ .

[10 marks]

- (b) Find the inverse of the following matrix

$$\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 2 & 7 & 7 \end{pmatrix}$$

[10 marks]

### QUESTION 2

2. The demand functions for two products are

$$p = 12 - 2x \quad \text{and} \quad q = 20 - y$$

where  $p$  and  $q$  are the respective prices for each product, and  $x$  and  $y$  are the respective amounts of each sold.

Suppose the joint cost function of these products is

$$C(x, y) = x^2 + 2xy + 2y^2$$

Find the revenue function and the profit function. Determine the prices and amounts that will maximize profit. What is the maximum profit? [20 marks]

### QUESTION 3

3. (a) Find the sum of the series

$$-1 + 2 + 5 + \dots + 56$$

[5 marks]

- (b) A new car costing E10000 depreciates 10% of its value each year. How much is the car worth at the end of six years? [5 marks]

- (c) How long will it take E10000 to grow to E12000 if it is invested at 9% compounded monthly? [5 marks]

- (d) What is the value of an annuity at the end of 10 years if E1000 is deposited each year into an account earning 8% compounded annually? [5 marks]

QUESTION 4

4. (a) Zanele wishes to purchase a computer in 5 years time. She figures that she will then need E10 000. What sum must she invest at the end of each quarter in a fund paying 16% compounded quarterly in order to accumulate the price of the computer? [10 marks]
- (b) A subscription share at a teachers' cooperative pays an annual interest rate of 8% compounded monthly. To what amount will payments of E30 made at the end of each month accumulate at the end of 3 years? [10 marks]

QUESTION 5

5. Solve the following minimization Linear Programming problem by maximizing the Dual. [20 marks]

$$\begin{aligned} \text{minimize } C &= 16x_1 + 45x_2 \\ \text{subject to } 2x_1 + 5x_2 &\geq 50 \\ x_1 + 3x_2 &\geq 27 \\ x_1, x_2 &\geq 0 \end{aligned}$$

QUESTION 6

6. A company manufactures two products X and Y. Each product has to be processed in three departments: welding, assembly and painting. Each unit of X spends 2 hours in the welding department, 3 hours in assembly and 1 hour in painting. The corresponding times for a unit of Y are 3, 2 and 1 respectively. The man-hours available in a month are 1500 for the welding department, 1500 in assembly and 550 in painting. The contribution to profits and fixed overheads are E100 for product X and E120 for product Y. Formulate the appropriate linear programming problem and solve it graphically to obtain the optimal solution for the maximum contribution. [20 marks]

QUESTION 7

7. (a) Consider a transportation problem in which the cost, supply and demand values are presented in the following table. The sources are the factories A, B and C and the destination are the warehouses 1, 2, 4 and 4.

	1	2	3	4	Supply
A	12	13	4	6	500
B	6	4	10	11	700
C	10	9	12	4	800
Demand	400	900	200	500	

- i. Is this a balanced transportation problem? [2 marks]
- ii. Find the initial basic feasible solution using the North-West Corner rule. [6 marks]
- (b) Use any method to solve the following system of equations 12 marks

$$2x + 3y + z = 2$$

$$-x + 2y + 3z = -1$$

$$-3x - 3y + z = 0$$