UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

BSc. II

TITLE OF PAPER	:	ELEMENTARY QUANTITATIVE TECHNIQUES II
COURSE NUMBER	:	MS 012
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(i) State the type of roots that the equation $\frac{1}{x} + \frac{1}{x-1} = 2$ has. [7]

(ii) The equation $x^2 - 2x + 1 = p(x - 3)$ has equal roots. Find the possible values of p. [7]

(iii) Solve the simultaneous equations $x + 2y = 7$ and $x^2 - 4x + y^2 = 1$. [6]

QUESTION 2

(i) Factorise the expression $x^3 + 3x^2 - 6x - 8$ [6]

(ii) Solve the equation $2x^3 - 3x^2 - 8x + 12 = 0$ [8]

(iii) Given $f(x) = \cos^4(2x + 1)$, find $f''(x)$. [6]

QUESTION 3

Evaluate the following limits

(i) $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$ [4]

(ii) $\lim_{x \to 0-} \frac{1}{x^3}$ [4]

(iii) $\lim_{x \to \infty} \frac{5x^2 + 3x + 2}{4x^2 + 7}$ [4]

(iv) $\lim_{x \to \infty} \left(e^{-x} + 1\right)$ [4]

(v) $\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$ [4]

QUESTION 4

a) Use the limit definitions of the derivative to find $f'(x)$ if $f(x) = \sqrt{x+1}$ [4]

b) Find $f'(x)$ for each of the following functions

(i) $f(x) = 2x^7 + 4x^2 + 3$ [3]

(ii) $f(x) = e^{4x+2} + x\cos x$ [3]

(iii) $f(x) = (x + 4\sin x)^{10}$ [3]

(iv) $f(x) = \frac{x+4}{x^2+1}$ [3]

QUESTION 5

a) Evaluate the following integrals

(i) $\int (5x^6 - x^2 + 6)dx$ [3]

(ii) $\int e^{2x+5}dx$ [3]

(iii) $\int \frac{2x+3}{x^2+3x}dx$ [3]

(iv) $\int_0^{\pi} \cos(3x+5)dx$ [4]

b) Find the area enclosed by the curve $f(x) = -x^2 + x + 2$ and the x-axis. [7]

QUESTION 6

a) Sketch the curve $f(x) = x^2 - 4x + 3$ by considering x and y intercepts, turning points, intervals of decrease and increase. [10]

b) Show that the function $f(x) = x^3 + x^2 + 5x + 6$ is always increasing. [4]

c) If $R = \frac{v^2}{4} + \frac{500}{v}$, find the value of v for which R is minimum. [6]

QUESTION 7

a) Given that $y = \cos(e^x)$, show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} + ye^{2x} = 0$. [7]

b) If $x - 2$, $x - 1$ and $3x - 5$ are the first three terms of a geometric progression, find

(i) the possible values of x [4]

(ii) the common ratio for each of the possible geometric progression. [4]

(iii) Solve the equation $\log_5 x = 1 - \log_5(x - 4)$ [5]