

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007/8

BASS I

TITLE OF PAPER : ELEMENTARY QUANTITATIVE TECHNIQUES II

COURSE NUMBER : MS 012

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF  
SEVEN QUESTIONS.  
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL  
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

- a) What type of roots does the equation  $2x^2 = px + p^2$  have [4]
- b) The equation  $x^2 - 2x + 1 = p(x - 3)$  has equal roots. Find the possible values of  $p$ . [6]
- c) Solve the equation  $2x^3 - 3x^2 - 8x + 12 = 0$  [10]

QUESTION 2

- a) Evaluate the following limits
- (i)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$  [5]
- (ii)  $\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 1}{x^2 + 4}$  [5]
- b) Use the limit definition of the derivative to find  $f'(x)$  if  $f(x) = \sqrt{x + 1}$  [10]

QUESTION 3

- a) Find  $f'(x)$  for each of the following functions
- (i)  $f(x) = x^5 + 2x^3 + x + 1$  [3]
- (ii)  $f(x) = (x^2 + 5x)^7$  [4]
- (iii)  $f(x) = e^{2x} \sin x + \cos 3x + \ln(x^2 + 1)$  [5]
- b) If  $\frac{A}{x} + Bx$ , where  $A$  and  $B$  are constants, show that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$$

[8]

QUESTION 4

a) Evaluate the following integrals

(i)  $\int_0^1 (3x^4 + 5x^3 + x + 1)dx$  [4]

(ii)  $\int \sqrt{x+1}dx$  [5]

(iii)  $\int \cos(3x+2)dx$  [4]

b) Find the area enclosed by the curve  $y = -x^2 + 9$  and the  $x$ -axis. [7]

QUESTION 5

a) For the function  $f(x) = x^2 - 4x + 3$ , find stationary points, intervals of decrease and increase. [8]

b) Make a rough sketch of the curve  $f(x) = x^3 - 6x^2 + 9x + 1$  by considering  $x$  and  $y$  intercepts, turning points, intervals of decrease and increase. [12]

### QUESTION 6

a) A piece wire 48cm long is divided into two parts. One part is formed into the shape of a circle of radius  $r$  cm while the other part is formed into a square of side  $x$  cm.

(i) Show that  $r = \frac{24-2x}{\pi}$  [4]

(ii) Find an expression in terms of  $x$  for the total area  $A$  of the two shapes and hence calculate the value of  $x$  for which  $A$  is a minimum. [6]

b) The cost of making  $x$  articles per day is  $E(\frac{1}{2}x^2 + 50x + 50)$  and the selling price of each one is  $E(80 - \frac{1}{4}x)$ . Find

(i) the daily profit in terms of  $x$  [5]

(ii) the value of  $x$  to give the maximum profit. [5]

### QUESTION 7

a) The first three terms of an  $AP$  are  $x$ ,  $2x + 1$ , and  $5x - 1$ . Find the value of  $x$  and the sum of the first 10 terms. [8]

b) Show that  $x + 1$ ,  $x + 3$  and  $x + 5$  cannot be three consecutive terms of a  $GP$ , whatever the value of  $x$ . [6]

c) Solve the equation  $5^{x^2+1} = 6$ . [6]