

UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2007/8

B.A.S.S. I / D.COM I

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| <u>TITLE OF PAPER</u> | : | INTRODUCTORY MATHEMATICS FOR BUSINESS |
| <u>COURSE NUMBER</u> | : | MS 101 AND IDE MS101 |
| <u>TIME ALLOWED</u> | : | THREE (3) HOURS |
| <u>INSTRUCTIONS</u> | : | 1. THIS PAPER CONSISTS OF <u>SEVEN</u> QUESTIONS. 2. ANSWER ANY <u>FIVE</u> QUESTIONS 3. USEFUL FORMULAE ARE PROVIDED AT THE END OF THE QUESTION PAPER. |
| <u>SPECIAL REQUIREMENTS</u> | : | NONE |

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Use synthetic division to find the quotient and remainder when $-x^4 + 5x^2 - 3x^3$ is divided by $x + 3$. [6 marks]
- (b) The polynomial $Ax^3 + 3x^2 + Bx - 12$ has $(x + 3)$ as a factor. When the polynomial is divided by $x + 1$ the remainder is -6 . Find the values of A and B . [6 marks]
- (c) Find all the real roots of the polynomial $x^4 - x^3 - 19x^2 + 49x - 30 = 0$ [8 marks]

QUESTION 2

2. (a) Zanele wants to buy a new computer in three years' time that will cost E5000.
- (i) How much should he deposit now, at 5% interest compounded annually to give the required E5000 in three years? [3 marks]
- (ii) If he only has E4000 available to deposit now, what annual interest rate is required for it to increase to E5000 in three years? [4 marks]
- (b) Find the annual interest rate required to double a certain amount of money if the interest is compounded monthly for 10 years. [4 marks]
- (c) How many years will be needed for E5000 to increase to E25000 at 5% interest compounded continuously? [5 marks]
- (d) Solve the following equation

$$\log(y + 1) = \log(y - 7) + \log 4$$

[4 marks]

QUESTION 3

3. (a) Prove the trigonometric identity

$$\csc 2x - \cot 2x = \tan x$$

[4 marks]

- (b) Solve the trigonometric equation

$$\cos 2x + \sin x = 0$$

giving all solutions between 0° and 360° .

[6 marks]

- (c) Convert the decimal 2.7121212 into a common fraction

[5 marks]

- (d) Find the sum of the following progressions

$$2 + 5 + 8 + 11 + \dots + 1001$$

[5 marks]

QUESTION 4

4. (a) Find the 15th term in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^{21}$ [6 marks]

- (b) Find the term involving x^{-9} in the expansion of $\left(2\sqrt{x} - \frac{1}{2x}\right)^{18}$. [7 marks]

- (c) Write out the first four terms in the expansion of $\frac{1}{\sqrt{1-x}}$ and use this expansion to estimate $\frac{1}{\sqrt{0.99}}$ correct to four significant figures. [7 marks]

QUESTION 5

5. (a) Calculate $A^T B$ if the matrices A and B be given by

$$A = \begin{pmatrix} 1 & -2 \\ 4 & 3 \\ 6 & 5 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 5 \\ -2 & 4 \\ 1 & 3 \\ 3 & 1 \end{pmatrix}$$

[6 marks]

- (b) use Cramer's rule to solve the following system of equations

$$\begin{aligned} 2x + 3y + z &= 2 \\ -x + 2y + 3z &= -1 \\ -3x - 3y + z &= 0 \end{aligned}$$

[14 marks]

QUESTION 6

6. (a) Write the equation of the circle with centre $(2,-7)$ and which passes through the point $(-2,-4)$ [4 marks]
- (b) Find the centre and the radius of the circle given by $x^2 + y^2 + 3x - 5y - \frac{1}{2} = 0$ [6 marks]
- (c) Find the equation of a straight line passing through the intersection of $3x - y = 9$ and $x + 2y = -4$, and is perpendicular to $3 = 2y + 8x$ [6 marks]
- (d) Find an equation of the line parallel to the line $y = -2x + 3$ and which passes through the point $(2,5)$. [4 marks]

QUESTION 7

7. (a) Express the following expressions in the complex form $a + bi$

(i) $(2 - 3i)(1 + 3i)$ [4 marks]

(ii) $\frac{5 - 2i}{2 - 3i}$ [4 marks]

(iii) $\sqrt{2}(\cos 225 + i \sin 225)$ [4 marks]

(b) Solve the quadratic equation

$$z^2 - 3z + 3 - i = 0$$

[8 marks]

END OF EXAMINATION

Useful Formulas

1. $\sin^2 \theta + \cos^2 \theta = 1$

2. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

3. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

4. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

5. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

6. $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

7. $\sin 2A = 2 \sin A \cos A$

8. $\cos 2A = \cos^2 A - \sin^2 A$

| Degrees | 0° | 30° | 45° | 60° | 90° |
|---------------|-----------|----------------------|----------------------|----------------------|------------|
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan \theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | |