UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008

B.A.S.S. I / D.COM I

TITLE OF PAPER	:	CALCULUS FOR BUSINESS AND SOCIAL SCIENCE
COURSE NUMBER	:	MS 102 AND IDE MS102
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
		3. SHOW ALL THE RELEVANT WORKING
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

      (i) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{x^2 - x - 2}$ [4 marks]

      (ii) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$ [4 marks]

    (b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions.

      (i) $f(x) = \sqrt{x+1}$ [6 marks]

      (ii) $f(x) = x^2 - x$ [6 marks]

QUESTION 2

2. Find the derivatives of the following functions

    (a) $y = x^2 \sin 2x$ [5 marks]

    (b) $y = \dfrac{x}{7 - 3x}$ [5 marks]

    (c) $y = (x^3 + 4x)^7$ [5 marks]

    (d) $y = x^{\cos x}$ [5 marks]

QUESTION 3

3. Find the following integrals

    (a) $\displaystyle\int_0^2 \left(6x^2 - 4x + 5\right)\, dx$ [5 marks]

    (b) $\displaystyle\int \dfrac{x}{\sqrt{x^2 + 1}}\, dx$ [5 marks]

    (c) $\displaystyle\int \dfrac{2x - 1}{x^2 - 3x + 2}\, dx$ [5 marks]

    (d) $\displaystyle\int \dfrac{\ln x}{x^2}\, dx$ [5 marks]

QUESTION 4

4. A Radio manufacturer determines that in order to sell x units of a new stereo, the price per unit, in dollars, must be $p = 1000 - x$. The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 2x$.

   (a) Find the total Revenue function $R(x)$. [2 marks]

   (b) Find the total Profit function $P(x)$. [2 marks]

   (c) How many units must the company produce and sell in order to maximize profit? [8 marks]

   (d) What is the maximum profit? [4 marks]

   (e) What price per unit must be charged in order to make this maximum profit? [4 marks]

QUESTION 5

5. (a) Find the area of the region enclosed by the parabolas

$$y = x^2 \quad \text{and} \quad y = 2x - x^2$$

[8 marks]

   (b) If the marginal cost of producing x units is given by

$$C'(x) = 0.3x^2 + 2x$$

   and the fixed cost is E2000, find

   (i)   the cost function $C(x)$. [3 marks]

   (ii)   the cost of producing 20 units [3 marks]

   (c) Find the consumers' surplus at a price level of E8 for the price-demand equation

$$p = D(x) = 20 - 0.05x$$

[6 marks]

QUESTION 6

6. (a) Find the equation of the tangent to the curve

$$y = x^4 - 6x^3 + 13x^2 - 10x + 5$$

at the point where $x = 1$ [8 marks]

(b) You are required to make a box with an open top from a 8 metre square sheet of cardboard by cutting smaller squares out of each corner of the sheet and folding up the edges.

Find the dimension of the box for which the volume is maximum. [12 marks]

QUESTION 7

7. (a) Given the function $f(x) = x^3 - 3x + 2$, find

   (i)   the y-intercept [1 mark]

   (ii)   stationary points [2 marks]

   (iii) intervals of increase and decrease [3 marks]

   (iv)   relative extrema [2 marks]

   (v)   inflection points [3 mark]

(b) Use all the information obtained in (a) to sketch a graph of the function. [4 marks]

(c) A company manufactures x printers per month. The marginal monthly profit (in Emalangeni) is given by

$$P'(x) = 165 - 0.1x$$

The company is manufacturing 1500 printers per month, but is planning to increase production. Find the total change in monthly profit if monthly production is increased to 1600 printers. [5 marks]

END OF EXAMINATION