UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2008

Dip.Comm II, IDE-Dip.Comm III

TITLE OF PAPER	:	QUANTITATIVE TECHNIQUES
COURSE NUMBER	:	MS 202
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Two catering companies, W and X, supply lunch to three schools, A, B and C. The cost matrix which also shows the number of lunch packs that can be supplied by each company and those required by each school, is given in the table below:

	DESTINATIONS			
Sources	A	B	C	Availabilities
W	4	2	2	500
X	6	3	5	300
Demands	200	400	200	

(i) Is this a balanced problem? Explain. [2 marks]

(ii) Obtain an initial feasible solution using the north-west corner rule. [6 marks]

(b) Use the Gauss-Jordan method to solve the following linear system of equations

$$\begin{aligned} 2x + y - z &= 1 \\ 3x - 2y + z &= 2 \\ x + 2y - z &= 2 \end{aligned}$$

[12 marks]

QUESTION 2

2. Namboard (PTY) LTD produces two types of fruit packages. Package A contains 20 peaches, 15 apples and 10 pears. Package B contains 10 peaches, 30 apples and 12 pears. Namboard has 40 000 peaches, 60 000 apples and 27 000 pears available for packaging. The profit on package A is E2.00 and the profit on B is E2.50. It is assumed that all fruits packaged can be sold. Formulate the problem as a linear programming problem and use the **graphical method** to determine the number of packages of type A and B that should be prepared to maximize the profit? [20 marks]

QUESTION 3

3. (a) Solve the following Linear Programming problem using the Simplex Method

$$\begin{aligned} \text{maximize } P = x_1 &+ 1.5x_2 \\ \text{subject to } 2x_1 + 4x_2 &\le 12 \\ 3x_1 + 2x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

[10 marks]

(b) The demand functions for two products are

$$p = 100 - 2x \ , \qquad q = 76 - y$$

where p and q are the respective prices (in thousands of Emalangeni) and x and y are the respective amounts (in thousands of units) of each product sold. If the joint cost function is given by

$$C(x, y) = 3x^2 + 2xy + 2y^2 + 55$$

determine the quantities and prices that maximize profit. What is the maximum profit? [10 marks]

QUESTION 4

4. (a) What principal amount invested today will amount to E2000 after 2 years if the rate of simple interest is 10%? [6 marks]

(b) A debt of E4000 at an interest rate of 10.5% is due in 10 months. The debtor pays E500 at the end of two months, and E1200 at the end of 5 months. Find the balance due in one year under the Merchant's rule. [7 marks]

(c) A subscription share at a workers' union pays an annual interest rate of 7.5% compounded monthly. To what amount will payments of E30 made at the end of each month accumulate at the end of 3 years? [7 marks]

QUESTION 5

5. (a) If I invest E1000 now at 11% compounded monthly, how long do I have to wait for my money to double? [6 marks]

   (b) A student is given E6000 three months before starting university. He invests this principal in a savings plan that pays 7% compounded quarterly. How much can he withdraw at the beginning of each quarter for the next three years so that the final withdrawal depletes the savings account? [7 marks]

   (c) For how many months must E60 be invested in a building society subscription share before an amount of E2000 can be withdrawn? The interest rate is 8%, compounded monthly. [7 marks]

QUESTION 6

6. (a) Find the inverse of the following matrix

$$A = \begin{pmatrix} 7 & 0 & -3 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

[10 marks]

   (b) Steers sells 1000 hamburgers, 600 cheeseburgers, and 1200 milk-shakes in a week. The price of a hamburger is 45c, a cheeseburger 60c, and a milk-shake 50c. The cost to the food outlet of a hamburger is 38c, a cheeseburger 42c, and a milk-shake 32c. Find the company's profit for the week. [10 marks]

QUESTION 7

7. (a) A firm produces two types of calculators, x thousand of type A and y thousand of type B per year. If the revenue and cost equations for the year are (in millions of dollars)

$$\begin{aligned} R(x,y) &= 2x + 3y \\ C(x,y) &= x^2 - 2xy + 2y^2 + 6x - 9y + 5 \end{aligned}$$

determine how many of each type of calculator should be produced per year to maximize profit? What is the maximum profit? [10 marks]

(b) Use the method of Lagrange Multipliers to optimize

$$C = 8x^2 - xy + 12y^2$$

subject to

$$x + y = 42$$

[10 marks]