UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2008/9

B.A.S.S I

TITLE OF PAPER   :   ELEMENTARY QUANTITATIVE TECHNIQUES

COURSE NUMBER   :   MS 011

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF

       SEVEN QUESTIONS.

      2.   ANSWER ANY FIVE QUESTIONS.

      3.   CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Express $\log 45$ in terms of $\log 3$ and $\log 5$. [3 marks]

   (b) Solve for x in the equation

   $$3^{3x+2} = 4^{4x-3}$$

   and give your answer correct to 4 decimal places. [5 marks]

   (c) Find x and y if

       i. $y = 27x$ and $\log_3 x + \log_3 y = 7$, [6 marks]

       ii. $y = 3\log_4 4$ and $y = \log_4 16x + 4$. [6 marks]

QUESTION 2

2. (a) Simplify fully;
   $$\frac{7m^6 n^3 p^2}{5r^3 q} \div \frac{6nm^4 p^9}{2q^3 r^7 t}$$

   [8 marks]

   (b) Express as a single fraction in its simplest form;

   $$\frac{x+2}{x-5} - \frac{x-4}{x+2}$$

   [6 marks]

   (c) Without using a calculator simplify the expression $\dfrac{2+\sqrt{3}}{5-\sqrt{7}}$. [6 marks]

QUESTION 3

3. (a) Prove the trigonometric identity
   $$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 \equiv 2$$
   [5 marks]

   (b) Given that $X = x\sin\theta + y\cos\theta$ and $Y = x\cos\theta - y\sin\theta$, show that $X^2 + Y^2 = x^2 + y^2$. [5 marks]

   (c) Solve the following trigonometric equations in the given ranges:

       i. $\cot 2\theta = 2$; $0 \le \theta \le 360°$ [5 marks]

       ii. $3\sin 2\theta - \cos\theta = 0$; $0 \le \theta \le \pi$ [5 marks]

QUESTION 4

4. (a) When $x^3 + 3x^2 + ax + b$ is divided by $(x+1)$ the remainder is 5, and when it is divided by $(x-2)$ the remainder is 8. Find a and b. [10 marks]

   (b) Find the quotient and the remainder of the division;

   $(x^3 + 4x - 3) \div (x - 2)$. [10 marks]

QUESTION 5

5. (a) A line L has equation $y = 3x + 4$. Find the equation of the each of the following lines.

   i. parallel to L, through $(1, 2)$. [5 marks]

   ii. perpendicular to L, through $\left(2, \dfrac{1}{3}\right)$. [5 marks]

   (b) A function is defined by $f : x \to (x-1)^2 - 2$ for $x \geq 1$.

   i. Find the range of f. [2 marks]

   ii. Find the inverse function f^{-1}, stating its domain and range. [8 marks]

QUESTION 6

6. (a) The area of a square less the side of the square is $\dfrac{143}{4}$. Find the side. [7 marks]

   (b) A population is decreasing at 2% each year. How long will it take for the population to halve? [7 marks]

   (c) The volume of a rectangular box is 120. The box has a base whose length l is twice the width. Show that its height h is given by $h = \dfrac{240}{l^2}$. [6 marks]

QUESTION 7

7. (a) If $E200$ is borrowed for 3 years at a 12% simple interest rate find the interest and total amount due at the end of 3 years. [7 marks]

   (b) Suppose that $E3000$ is invested in an account paying 10% interest. Find the amount at the end of 7 years if the interest is

   i. compounded annually, [6 marks]

   ii. compounded semi-annually. [7 marks]