UNIVERSITY OF SWAZILAND

SUPPLEMENTARTY EXAMINATION 2008/9

B.A.S.S I

TITLE OF PAPER   :   ELEMENTARY QUANTITATIVE TECHNIQUES

COURSE NUMBER   :   MS 011

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF

                             SEVEN QUESTIONS.

                   2.   ANSWER ANY FIVE QUESTIONS.

                   3.   CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Express $\log 108$ in terms of $\log 2$ and $\log 3$. [3 marks]

   (b) Solve for x in the equation

   $$2^{2x-3} = 3^{3x+2}$$

   and give your answer correct to 4 decimal places. [5 marks]

   (c) Solve the following simultaneous equations for x and y;

       i. $y = \log_2(x^2 + 1)$ and $y = \log_2(x + 1) + 1$. [6 marks]

       ii. $xy = 27$ and $\log_3 x = 1 + \log_3 y$, [6 marks]

QUESTION 2

2. (a) Express as a single fraction in its simplest form;

   $$\frac{x+3}{x-1} - \frac{x-3}{x+4}$$

   [6 marks]

   (b) Without using a calculator simplify the expression $\dfrac{\sqrt{3} + \sqrt{2}}{3 - \sqrt{5}}$. [6 marks]

   (c) Simplify fully;

   $$\frac{3m^5p^3n^4}{5q^2r^2} \div \frac{8m^2n^3p^4}{18r^9u^2q}$$

   [8 marks]

QUESTION 3

3. (a) Prove the trigonometric identity

   $$\sin^3\theta + \cos^3\theta \equiv (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$$ [5 marks]

   (b) Use trigonometric identities to write

   $$\cot^2\theta + 5\operatorname{cosec}^2\theta + 2$$

   in terms of a single trigonometric function. [5 marks]

   (c) Solve the following trigonometric equations in the given ranges:

       i. $\cos(\theta + 11°) = 0.5$; $0 \le \theta \le 360°$ [5 marks]

       ii. $2\cos 2\theta + \cos^2\theta = 1$; $0 \le \theta \le \pi$ [5 marks]

QUESTION 4

4. (a) If $(x + 5)$ and $(x + 2)$ are both factors of $x^3 + 3x^2 + ax + b$, find a and
    b. [10 marks]

   (b) Find the quotient and the remainder of the division;
    $(x^3 - 3x^2 + 4x + 1) \div (x - 2)$. [10 marks]

QUESTION 5

5. (a) Find the equation of each of the following lines;

     i. through $(2, 1)$ and parallel to the line through $(1, 3)$ and $(0, -1)$.
        [5 marks]

     ii. through $(4, 1)$ and perpendicular to $y = 2x - 3$. [5 marks]

   (b) Let $f(x) = (x - 2)^2 - 1$ for $x \geq 2$.

     i. Find the range of f. [2 marks]

     ii. Find the inverse function f^{-1}, stating its domain and range. [8 marks]

QUESTION 6

6. (a) The area of a circle of radius r less its radius is $\dfrac{2}{\pi}$. Find r. [7 marks]

   (b) A population is increasing at 5% each year. How long will it take for the
    population to trebble? [7 marks]

   (c) Let r be the radius of the cross-section of a cylinder whose volume equals
    four times its height. Show that $r = \dfrac{2}{\sqrt{\pi}}$. [6 marks]

QUESTION 7

7. (a) Suppose that $E2000$ is invested in an account paying 14% interest. Find
    the amount at the end of 5 years if the interest is

     i. simple, [6 marks]

     ii. compounded monthly. [7 marks]

   (b) Find the time required to trebble a certain amount that is invested in an
    account offering 11.5% interest compounded daily. [7 marks]