

UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2009

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS
3. SHOW ALL THE RELEVANT WORKING

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow \infty} \frac{2x^2 - 2x + 1}{x^2 - x - 2}$ [4 marks]

(ii) $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$ [4 marks]

(b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions.

(i) $f(x) = \sqrt{x+1}$ [6 marks]

(ii) $f(x) = \frac{1}{x+1}$ [6 marks]

QUESTION 2

2. Find the derivatives of the following functions

(a) $y = \sin^5(x^3 + \ln x + e^{x^2})$ [5 marks]

(b) $y = \frac{x+1}{2-3x}$ [5 marks]

(c) $y = (x^3 + 4x + e^{2x})^5$ [5 marks]

(d) $y = x^{\sin x}$ [5 marks]

QUESTION 3

3. Find the following integrals

(a) $\int (x^3 + 2)^2 dx$ [5 marks]

(b) $\int \sqrt{4x+9} dx$ [5 marks]

(c) $\int x^7 \ln x dx$ [5 marks]

(d) $\int \frac{11-5x}{x^2+x-2} dx$ [5 marks]

QUESTION 4

4. A Radio manufacturer determines that in order to sell x units of a new stereo, the price per unit, in dollars, must be $p = 1000 - x$. The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 2x$.
- (a) Find the total Revenue function $R(x)$. [2 marks]
- (b) Find the total Profit function $P(x)$. [2 marks]
- (c) How many units must the company produce and sell in order to maximize profit? [8 marks]
- (d) What is the maximum profit? [4 marks]
- (e) What price per unit must be charged in order to make this maximum profit? [4 marks]

QUESTION 5

5. For the graph of the function $f(x) = x^3 + 3x^2 - 9x + 15$
- (a) Find its y -intercept. [2 marks]
- (b) Find the interval(s) where $f(x)$ is increasing and where $f(x)$ is decreasing. [5 marks]
- (c) Use information obtained in part (b) to find its point(s) of relative maximum and relative minimum. [3 marks]
- (d) Find the interval(s) where the function is concave up and where it is concave down. [3 marks]
- (e) What are its point(s) of inflection? [2 marks]
- (f) Sketch the graph of the function $f(x)$ using information obtained in parts (a)-(e) [5 marks]

QUESTION 6

6. The demand equation for a product is $p = D(x) = 300 - 0.03x^2$, and the supply equation is $p = S(x) = 0.09x^2$, where p is the price and x is the quantity.

- (a) Find the equilibrium price p^* and quantity x^* . [6 marks]
- (b) Find the consumer surplus CS. [7 marks]
- (c) Find the producer surplus PS. [7 marks]

QUESTION 7

7. (a) Find $f(x)$ given $f'(x) = 7x^4 - 9x + 5$ and $f(0) = 5$. [6 marks]
- (b) A manufacturer of mountain bikes has marginal cost function

$$C'(x) = \frac{600}{0.3x + 5}$$

where x is the quantity of bicycles produced. If the fixed cost in producing the bicycles is E2000, find the total cost to produce 30 bicycles. [7 marks]

- (c) Find the area between the curves $y = x^2$ and $y = 3x$. [7 marks]

END OF EXAMINATION