

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2009

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Evaluate the following limits:

(i) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 9}$ [3 marks]

(ii) $\lim_{x \rightarrow \infty} \frac{5 - 4x^2 + 3x}{5x^2 + 4}$ [3 marks]

(b) For what values of c is the function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 3 \\ 2cx & \text{if } x \geq 3 \end{cases}$$

continuous at every value of x ? [6 marks]

(c) Find the points, if any, at which each of the following functions is discontinuous:

(i) $f(x) = \frac{x^3 - 1}{x^2 - 1}$ [3 marks]

(ii)

$$g(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ 6 & \text{if } x = 4 \end{cases}$$

[5 marks]

QUESTION 2

(a) Use the limit definition of the derivative to find $\frac{dy}{dx}$ of each of the following functions:

(i) $y = \frac{1}{x}$ [6 marks]

(ii) $y = -3x^2 + 5x$ [6 marks]

(b) Find the derivatives, $\frac{dy}{dx}$, of the following functions

(i) $y = \sin^3 5x$ [4 marks]

(ii) $y = \frac{x+1}{(2x+7)^{10}}$ [4 marks]

QUESTION 3

(a) For the curve given by $y = x^3 - 6x^2 + 9x + 10$ find the points where the curve assumes a

(i) a local maximum [2 marks]

(ii) a local minimum [2 marks]

(iii) a point of inflexion [2 marks]

(b) Find the intervals where the curve is

(i) rising [2 marks]

(ii) falling [2 marks]

(iii) concave up [3 marks]

(iv) concave down [3 marks]

(c) Using the information in (a) and (b), draw a rough sketch of the curve [4 marks]

QUESTION 4

(a) The total population of a town after t years is given by $P(t) = 200t^2 + 1500$.

(i) Find the total population of the town after 4 years. [1 mark]

(ii) Find the instantaneous rate of change in the population after 4 years and interpret your results. [4 marks]

(b) Find the equation of the tangent to the curve

$y = x^2 + 5x + 5$ at $x = -1$. [3 marks]

(c) A company manufactures cordless telephones and finds that its cost function $C(x)$ and revenue function $R(x)$ in Emalangeneni are given by

$$C(x) = 400\sqrt{x} + 700 \quad \text{and} \quad R(x) = 500\sqrt{x} + 300$$

respectively, where x is the number of telephones produced.

(i) Find the marginal cost function. [2 marks]

(ii) Find the marginal cost when 100 telephones have been produced and interpret your answer. [3 marks]

(iii) Find the profit function $P(x)$. [2 marks]

(iv) Find the profit when 100 telephones are sold. [2 marks]

(v) Find the marginal profit when 100 telephones have been sold and interpret your answer. [3 marks]

QUESTION 5

Evaluate the following integrals

(a) $\int (x + 1) (\sqrt{x^2 + 2x}) dx$ [5 marks]

(b) $\int (x^2 + 1) e^x dx$ [5 marks]

(c) $\int \left(\frac{x + 2}{x^2 - 2x - 3} \right) dx$ [5 marks]

(d) $\int \sin^3 x dx$ [5 marks]

QUESTION 6

(a) Find the area of the region bounded by the curves $y = 9 - x^2$ and $y = x^2 + 1$ [8 marks]

(b) The marginal profit, in thousands of Emalangeni, for producing and selling x items is given by

$$P'(x) = 600 - 5x.$$

(i) Determine the total-profit function, given that the total profit is 45000 when 100 units are produced and sold. [6 marks]

(ii) What is the change in total profit if the level of production changes from 50 to 55? [6 marks]

QUESTION 7

Given the demand function $D(x) = 25 - 0.001x^2$ and the supply function

$S(x) = 5 + 0.1x$, find

(a) the equilibrium price. [7 marks]

(b) the consumer's surplus at the market demand found in part (a). [7 marks]

(c) the producer's surplus at the market demand found in part (a). [6 marks]