UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2009

Dip.Comm II, IDE-Dip.Comm III

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :
1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Use the Gauss-Jordan method to find the inverse of the matrix;

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -3 & -4 \\ -1 & -2 & -1 \end{bmatrix}$$

[10 marks]

(b) A company has two interacting branches A and B. Branch A consumes E0.5 of own output and E0.2 of B output for every E1 it produces. Branch B consumes E0.6 of A output and E0.4 of its own output per E1 of output. The company wants to know how much each branch should produce per month in order to meet exactly a monthly external demand of E50,000 for A product and E40,000 for B product. [10 marks]

QUESTION 2

2. (a) Use the method of Lagrange multipliers to optimize the function;

$$f(x,y) = 9 - x^2 - y^2,$$
$$\text{subject to } x + y = 2$$

[10 marks]

(b) A company manufactures x ten-speed bicycles which sell for Ep and y three-speed bicycles which sell for Eq. The weekly demand and cost equations are as follows:

$$\begin{aligned} p &= 230 - 9x + y \\ q &= 130 + x - 4y \\ C(x,y) &= 200 + 80x + 30y \end{aligned}$$

i. How many of each type of bicycle should be sold in order to maximize profit? [6 marks]

ii. At what prices should the bicycles be sold? [2 marks]

iii. What will the maximum profit be? [2 marks]

QUESTION 3

3. (a) Use Gaussian elimination to solve the linear system;

$$\begin{aligned} x + 3y + 3z &= 16 \\ x + 4y + 3z &= 18 \\ 2x + 7y + 7z &= 37 \end{aligned}$$

[10 marks]

(b) Find all the local maximum, minimum and saddle points of the following function

$$f(x, y) = 2x^3 - 6xy + 3y^2 + 6x - 18y$$

[10 marks]

QUESTION 4

4. (a) A furniture company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hours in the painting department.

Each table takes 4 hours of carpentry and 2 hours in the painting department. Each chair requires 3 hours of carpentry and 1 hour in the painting department. During the current production period, 240 hours of carpentry time are available and 100 hours in painting is available. Each table sold yields a profit of E7; each chair produced is sold for a E5 profit.

Formulate the linear programming problem and use the graphical method to find the best combination of tables and chairs to manufacture in order to reach the maximum profit. [10 marks]

(b) A company manufactures two products, each requiring the use of three machines. The first machine can be used at most 70 hours; the second machine at most 40 hours; and the third machine at most 90 hours. The first product requires 2 hours on machine 1, 1 hour on machine 2, and 1 hour on machine 3; the second product requires 1 hour on each of machines 1 and 2 and, it needs 3 hours on machine 3. If the profit is E40 per unit for the first product and E60 per unit for the second product, how many units of each product should be manufactured to maximize profit?

[10 marks]

QUESTION 5

5. (a) A debt of E1200 is to be paid off by payments of E500 in 45 days, E300 in 100 days and a final payment of E436.92. Interest is at 11% and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made? [7 marks]

   (b) Sydney wishes to purchase a modest ocean going boat in 5 years time. He figures that he will need E170 000 then. What sum must he invest at the end of each quarter in a fund paying 12% compounded quarterly in order to accumulate the price of the boat? [7 marks]

   (c) How much should you deposit in an account paying 6% compounded semi-annually in order to be able to withdraw E1000 every 6 months for the next 3 years? [6 marks]

QUESTION 6

6. A trucking company has a contract to move 115 truckloads of sand per week between three sand-washing plants W,X and Y, and three destinations, A,B and C. Cost and volume information is given below. Compute the optimal transportation cost using the north-west corner rule.

From \ To	Project A	Project B	Project C	Supply
Plant W	5	10	10	35
Plant X	20	30	20	40
Plant Y	5	8	12	40
Demand	45	50	20	

[20 marks]

QUESTION 7

7. (a) Solve the following minimization Linear Programming problem by maximizing the Dual. [12 marks]

$$\begin{aligned} \text{minimize } & C = 16x_1 + 45x_2 \\ \text{subject to } & 2x_1 + 5x_2 \geq 50 \\ & x_1 + 3x_2 \geq 27 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(b) Steers sells 1000 hamburgers, 600 cheeseburgers, and 1200 milk-shakes in a week. The price of a hamburger is 45c, a cheeseburger 60c, and a milk-shake 50c. The cost to the food outlet of a hamburger is 38c, a cheeseburger 42c, and a milk-shake 32c.
Find the company's profit for the week. [8 marks]