
University of Swaziland



Final Examination, 2009/2010

Bass I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MS012

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

- (a) State the type of roots that the equation

$$\frac{1}{x} + \frac{1}{x-1} = 2$$

has. [7]

- (b)

- (i) Factorise the expression

$$x^3 + 3x^2 - 6x - 8. \quad [7]$$

- (ii) Hence, or otherwise, solve the equation

$$x^3 + 3x^2 - 6x - 8 = 0. \quad [6]$$

Question 2

Evaluate the following limits.

(a) $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$ [4]

(b) $\lim_{x \rightarrow 0^-} \frac{1}{x^3}$ [3]

(c) $\lim_{x \rightarrow \infty} \frac{5x^2 + 3x + 2}{4x^2 + 7}$ [4]

(d) $\lim_{x \rightarrow \infty} (e^{-x} + 1)$ [3]

(e) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$ [6]

Question 3

- (a) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x) = \sqrt{x+1}. \quad [8]$$

- (b) Find $f'(x)$ for each of the following functions.

(i) $f(x) = x^5 + 2x^3 + x + 1$ [3]

(ii) $f(x) = (x^2 + 5x)^{12}$ [3]

(iii) $f(x) = e^{2x} + \cos x + \ln(x^2)$ [3]

(iv) $f(x) = \frac{x^2}{x+2}$ [3]

Question 4

- (a) Evaluate the following integrals.

(i) $\int (5x^6 - x^2 + 6)dx$ [4]

(ii) $\int e^{2x+5}dx$ [4]

(iii) $\int \frac{2x+3}{x^2+3x}dx$ [4]

(iv) $\int_0^{30^\circ} \cos(3x)dx$ [4]

- (b) Find the area enclosed by the curve $f(x) = -x^2 + x + 2$ and the x -axis. [4]
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Question 5

- (a) A piece of wire 48cm long is divided into two parts. One part is formed into the shape of a circle of radius r cm while the other part is formed into a square of side x cm.

(i) Show that $r = \frac{24 - 2x}{\pi}$ [4]

- (ii) Find an expression in terms of x for the total area A of the two shapes, and hence calculate the value of x for which A is a minimum. [6]

- (b) The cost of making x articles per day is $\text{E}(\frac{1}{2}x^2 + 50x + 50)$ and the selling price of each article is $\text{E}(80 - \frac{1}{4}x)$. Find

(i) The daily profit in terms of x [4]

(ii) The value of x to give the maximum profit. [6]

Question 6

- (a) For the function $f(x) = x^3 - 6x^2 + 9x + 1$, find the critical points, stationary points, intervals of increase and decrease.

Sketch the graph of $f(x) = x^3 - 6x^2 + 9x + 1$. [12]

- (b) Show that the function $f(x) = x^3 + x^2 + 5x + 6$ is always increasing. [8]
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Question 7

The supply and demand equations for a certain commodity are:

$$p = S(q) = q^2 + 8q + 5, \quad \text{and}$$

$$p = D(q) = -q^2 - 7q + 1408,$$

where $0 \leq q \leq 32$, p is in Emalangeni and q is the number of units in thousands.

- (i) Find the equilibrium quantity and price. [6]
 - (ii) Find the consumer's surplus. [7]
 - (iii) Find the producer's surplus. [7]
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