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# University of Swaziland



Supplementary Examination, 2009/2010

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## Bass I

**Title of Paper** : Elementary Quantitative Techniques II

**Course Number** : MS012

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

**Question 1**

(a) The equation  $x^2 - 2x + 1 = p(x - 3)$  has equal roots.  
Find the possible values of  $p$ . [10]

(b) Solve the simultaneous equations

$$\begin{aligned}x + 2y &= 7 \\x^2 - 4x + y^2 &= 1.\end{aligned}$$

[5]

(c) Consider the graph of  $f(x)$  below.

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**Question 2**

(a) State the factor theorem. [4]

(b) Factorise the expression

$$x^3 + 3x^2 - 6x - 8. \quad [6]$$

(c) Solve the equation

$$2x^3 - 2x^2 - 8x + 12 = 0. \quad [10]$$

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### Question 3

Evaluate the following limits.

$$(a) \quad \lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} \quad [5]$$

$$(b) \quad \lim_{x \rightarrow 0^+} \frac{1}{x^2} \quad [3]$$

$$(c) \quad \lim_{x \rightarrow \infty} \frac{2x^4 + x^2 + 1}{x^5 + 3x} \quad [5]$$

$$(d) \quad \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} \quad [7]$$

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### Question 4

(a) Use the limit definition of the derivative to find  $f'(x)$  if  $f(x) = \sqrt{x}$ . [8]

(b) Find  $f'(x)$  for each of the following functions.

$$(i) \quad f(x) = 3x^4 + 2x^2 + 4 \quad [3]$$

$$(ii) \quad f(x) = (x + \sin^2 x)^6 \quad [3]$$

$$(iii) \quad f(x) = \frac{x + 2}{x^2 + 1} \quad [3]$$

$$(iv) \quad f(x) = \cos x \ln(x^2 + 1) \quad [3]$$

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### Question 5

Evaluate the following integrals.

$$(a) \quad \int (5x^7 + 3x^2 + 2) dx \quad [4]$$

$$(b) \quad \int e^{3x+4} dx \quad [5]$$

(c)  $\int_0^{\pi/2} \sin(2x + 3) dx$  [5]

(d)  $\int x e^x dx$  [6]

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**Question 6**

- (a) A rectangular box, with a lid, is made from thin metal. Its length is  $2x$  cm and its width is  $x$  cm. If the box must have a volume of  $72 \text{ cm}^3$ ,

- (i) Show that the area  $A \text{ cm}^2$  of the metal used is given by

$$A = 4x^2 + \frac{216}{x}, \quad [5]$$

- (ii) Find the value of  $x$  so that  $A$  is a minimum. [6]

- (b) The function  $y = x^3 + ax^2 - 7x - 1$  has a stationary value when  $x = 1$ .

Find

- (i) the value of  $a$  [4]  
(ii) the type and position of the stationary point. [6]
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**Question 7**

- (a) Sketch the curve  $f(x) = 2 + x - x^2$ . Find the area that is enclosed by  $f(x) = 2 + x - x^2$  and the  $x$ -axis. [8]

- (b) On graph paper, make a sketch of  $f(x) = x^3 - 6x^2$ . Mark clearly all stationary points, intercepts and points of inflexion on your graph. [12]
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