UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2010

B.A.S.S. I / D.COM I

TITLE OF PAPER   :   CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER   :   MS 102 AND IDE MS102

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF
                                    SEVEN QUESTIONS.
                               2.   ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

    (i) $\quad \lim\limits_{x \to -3} \dfrac{x+3}{9-x^2}$ [4 marks]

    (ii) $\quad \lim\limits_{x \to \infty} \dfrac{1-x^2-2x^3}{4x^3+x+2}$ [4 marks]

   (b) Use the **limit definition** of the derivative to find $f'(x)$ corresponding to the following functions

    i. $\quad f(x) = \dfrac{1}{1+x}$ [6 marks]

    ii. $\quad f(x) = 1 - 2x^2$ [6 marks]

QUESTION 2

2. Find the derivatives, $\dfrac{dy}{dx}$, of the following functions

    (a) $\quad y = e^{x^2+x+1}$ [5 marks]

    (b) $\quad y = x^3 \ln x^2$ [5 marks]

    (c) $\quad y = \sin^3(5x+1)$ [5 marks]

    (d) $\quad y = \dfrac{1}{(x^2+2x+1)^4}$ [5 marks]

QUESTION 3

3. (a) The Ministry of Works and Transport is planning to build a picnic park for motorists along the Mbabane-Manzini highway. The park is to be rectangular with an area of 5000 square metres and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing required for this job? How long and how wide should the park be for the fencing to be minimized? [10 marks]

   (b) Find the equation of the tangent to the curve
   $y = x^2 + 5x + 5$ at $x = -1$. [4 marks]

   (c) A company manufactures cordless telephones and finds that its cost function $C(x)$ and revenue function $R(x)$ in Emalangeni are given by

   $$C(x) = 400\sqrt{x} + 500 \quad \text{and} \quad R(x) = 500\sqrt{x} + 200$$

   respectively, where x is the number of telephones produced.

   (i) Find the profit function $P(x)$. [1 marks]

   (ii) Find the profit when 100 telephones are sold. [2 marks]

   (iii) Find the marginal profit when 100 telephones have been sold and interpret your answer. [3 marks]

QUESTION 4

4. (a) For the curve given by $y = x^3 - 6x^2 + 9x + 1$ find the points where the curve assumes a

    (i) a local maximum   [2 marks]

    (ii) a local minimum   [2 marks]

    (iii) a point of inflexion   [2 marks]

(b) Find the intervals where the curve is

    (i) rising   [2 marks]

    (ii) falling   [2 marks]

    (iii) concave up   [3 marks]

    (iv) concave down   [3 marks]

(c) Using the information in (a) and (b), draw a rough skecth of the curve   [4 marks]

QUESTION 5

5. Evaluate the following integrals

    (a) $\int \left(2x^3 + \dfrac{1}{x^2} - 3x + \dfrac{2}{3} \right) dx$ [5 marks]

    (b) $\int \left(x^2 + 1 \right) e^x dx$ [5 marks]

    (c) $\int \left(\dfrac{x+2}{x^2 - 2x - 3} \right) dx$ [5 marks]

    (d) $\int x\sqrt{x+1}\,dx$ [5 marks]

QUESTION 6

6. (a) Find the area of the region bounded by the parabola $y = x^2 + 3x + 2$ and the line $y = 2$ [8 marks]

   (b) The marginal profit, in thousands of emalangeni, for producing and selling x items is given by

   $$P'(x) = 600 - 5x.$$

   (i) Determine the total-profit function, given that the total profit is 45000 when 100 units are produced and sold.   [6 marks]

   (ii) What is the change in total profit if the level of production changes from 50 to 55?   [6 marks]

QUESTION 7

7. Given the demand function $p = D(x) = 25 - 0.001x^2$ and the supply function $p = S(x) = 5 + 0.1x$, find

   (a) the equilibrium price   [6 marks]

   (b) the consumer's surplus   [7 marks]

   (c) the producer's surplus   [7 marks]