

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2010

Dip.Comm II, IDE-Dip.Comm III

- TITLE OF PAPER : QUANTITATIVE TECHNIQUES
- COURSE NUMBER : MS 202
- TIME ALLOWED : THREE (3) HOURS
- INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.
- SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) For the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 2 & 7 & 7 \end{bmatrix}$$

determine $\text{Adj } A$ and use it to compute A^{-1} . [8 marks]

- (b) A company needs buy to two types (A and B) of cabinets for its newly established office. Each cabinet of type A costs $E20$, requires 8 square metres of floor space, and holds 12 cubic metres of files. Each cabinet of type B costs $E40$, requires 6 square metres of floor space, and holds 8 cubic metres of files. Only $E140$ is available for this purchase, though the company doesn't have to spent that much. The office has room for no more than 72 square metres of cabinets.

Formulate the linear programming problem and use the graphical method to determine how many of each type of cabinet the company should buy in order to maximize storage volume. [12 marks]

QUESTION 2

2. (a) Use Gaussian-Jordan method to solve the linear system;

$$\begin{aligned} x + 2y + 3z &= 3 \\ -3x + y + 5z &= -2 \\ 2x + 4y - z &= -1 \end{aligned}$$

[10 marks]

- (b) A company manufactures x units of product A and y units of product B per month. In terms of the respective prices per unit p and q , the demand equations are

$$\begin{aligned} x &= p + 2 \\ y &= q + 3 \end{aligned}$$

and the cost function for the company is

$$C(x, y) = 8x + 11y - 2x^2 - 2y^2 - 21$$

- i. Determine the quantities and prices that maximize profit. [8 marks]
- ii. What will the maximum monthly profit be? [2 marks]

QUESTION 3

3. (a) An economy is based on three industries: labour, transportation and food. Each E1 in labour requires 40c in transportation and 20c in food, while each E1 in transportation takes 50c in labour and 30c in transportation, and each E1 in food production uses 50c in labour, 5c in transportation, and 35c in food.

Let the demand for the current production period be E10,000 labour, E20,000 transportation, and E10,000 food.

Find the production schedule for the economy. [12 marks]

- (b) Find and classify all local extrema of the function

$$f(x, y) = x^3 + x^2y - y^2 - 4y. \quad [8 \text{ marks}]$$

QUESTION 4

4. (a) Solve the following linear programming problem by maximizing the dual.

$$\text{Minimize } C = 2x_1 + 2x_2,$$

$$\text{Subject to } x_1 + 2x_2 \geq 3,$$

$$3x_1 + 2x_2 \geq 5,$$

$$x_1, x_2 \geq 0.$$

[12 marks]

- (b) The demand, supply and cost (in E) values of a certain transportation problem are shown in the following table. The sources are the warehouses A, B, C and D, and the destinations are the depots W, X, Y and Z.

From \ To	W	X	Y	Z	Supply
A	3	7	5	5	38
B	2	5	3	4	19
C	2	4	4	5	16
D	4	9	4	6	23
Demand	25	29	21	21	

- i. Is this a balanced transportation problem?

Justify your answer.

[2 marks]

- ii. Find the initial basic feasible solution using the North-West corner rule.

[6 marks]

QUESTION 5

5. (a) Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

		Job				
		1	2	3	4	5
Person	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

[10 marks]

- (b) A computer games internet retailer has four favoured customers who each want a copy of the latest *FIFA 2010* computer game. The retailer has one copy available to it at each of two wholesalers in SA, and can get two further copies, one each from each of the two wholesalers in the UK. The costs of each possible allocation of copies (i.e wholesalers) to the customers are

Costs	Customer 1	Customer 2	Customer 3	Customer 4
Wholesaler 1	1	3	6	2
Wholesaler 2	5	2	3	4
Wholesaler 3	9	13	10	8
Wholesaler 4	7	12	8	5

Use the Hungarian Algorithm, find an optimal assignment of wholesalers to customers.

[10 marks]

QUESTION 6

6. (a) A debt of E1200 is to be paid off by payments of E500 in 45 days, E300 in 100 days and a final payment of E436.92. Interest is at 11% and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made? [7 marks]
- (b) Sydney wishes to purchase a modest ocean going boat in 5 years time. He figures that he will need E170 000 then. What sum must he invest the end of each quarter in a fund paying 12% compounded quarterly in order to accumulate the price of the boat? [7 marks]
- (c) How much should you deposit in an account paying 6% compounded semi-annually in order to be able to withdraw E1000 every 6 months for the next 3 years? [6 marks]

QUESTION 7

7. A lumber company ships pine flooring from its three mills, A_1 , A_2 and A_3 , to three building suppliers, B_1 , B_2 and B_3 . The table below shows the demand, availabilities and unit costs of transportation. Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost. [20 marks]

	B_1	B_2	B_3	Availability
A_1	3	3	2	25
A_2	4	2	3	40
A_3	3	4	3	31
Demand	30	30	36	