University of Swaziland

Final Examination, May 2011

BASS I

Title of Paper : Quantitative Techniques II

Course Number : MS012

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Use the limit definition (not formulas) to find the derivatives of the following

(i) $y = 2x^2 + 1$ [5]

(ii) $y = \sqrt{x}$ [5]

(b) Find the equation of the tangent to the curve

$$y = x^3 - 2x^2 + 2$$

at the point $(1, 1)$. [10]

Question 2

(a) Evaluate the following limit by first rationalizing the numerator.

$$\lim_{x \to 0} \left(\frac{\sqrt{1-x} - \sqrt{1+x}}{x} \right).$$ [8]

(b) Determine where the function $y = x^3 - 6x^2 + 9x + 1$ is increasing or decreasing and relative maximum or relative minimum. [12]

Question 3

Evaluate each of the following integrals.

(a) $\displaystyle\int \sqrt{3x+2}\,\mathrm{d}x$

(b) $\displaystyle\int \left(x^7 + 2x + \frac{3}{x^3} + \frac{4}{x}\right) \mathrm{d}x$

(c) $\displaystyle\int \frac{3x^2}{x^3+2}\,\mathrm{d}x$

(d) $\displaystyle\int \left(x^2+1\right)^5 x\,\mathrm{d}x$

[20]

Question 4

(a) Use the factor theorem to determine whether $x - 1$ is a factor of $x^3 - 3x^2 + 3x - 1$. [8]

(b) Evaluate the following definite integrals.

(i) $\displaystyle\int_{-2}^{2} \left(2x^3 + 3\right) \mathrm{d}x$

(ii) $\displaystyle\int_{0}^{2} \frac{x}{x^2+4}\,\mathrm{d}x$

[12]

Question 5

(a) Find the first derivatives of each of the following.

(i) $y = e^{\sqrt{1+x}}$

(ii) $y = \ln(\cos x)$

[8]

(b) Evaluate each of the following integrals.

(i) $\int \sec x \, \mathrm{d}x$

(ii) $\int x\sqrt{1+x} \, \mathrm{d}x$

[12]

Question 6

Find the area under the given graph but above the x-axis.

(a) $y = 5x - x^2 - 4$

(b) $y = 9 - x^2$

[20]

Question 7

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for each of the following.

(a) $y = x^2(x+2)$

(b) $y = \sin(3x+2)$

(c) $y = \dfrac{x+1}{x^2+3}$

(d) $y = e^{3x^2}$

[20]