University of Swaziland

Supplementary Examination, July 2011

BASS I

Title of Paper : Quantitative Techniques II

Course Number : MS012

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate the following limits

(i) $\lim_{x \to 1} \dfrac{x^3 - 1}{4x^3 - x - 3}$

(ii) $\lim_{x \to 1} \dfrac{2x^2 - 3x + 1}{x - 1}$

[10]

(b) Use the limit definition to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(i) $y = x^3 - x$

(ii) $y = \dfrac{1}{x}$

[10]

Question 2

(a) Find the equation of the tangent to the curve $y = x^3 - 3x + 3$ at the point $(2, 5)$. [10]

(b) Use the chain rule to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(i) $y = 2u^2, \quad u = x^2 - 1$

(ii) $y = t^2 + 1, \quad x = 3t + 4$

[10]

Question 3

(a) Find the area of the region lying above the x-axis and under the parabola

$$y = 4x - x^2. \qquad [10]$$

(b) Differentiate each of the following

(i) $y = \sin 2x$ [5]

(ii) $y = \dfrac{x-1}{x+2}$ [5]

Question 4

(a) Find all the relative maxima and relative minima of the function $y = x^4 - x^3 - x^2 + x$. [10]

(b) Evaluate the following integrals.

(i) $\displaystyle\int \frac{\cos x}{1+\sin x}\mathrm{d}x$ [5]

(ii) $\displaystyle\int \frac{1}{(9+x)^5}\,\mathrm{d}x$ [5]

Question 5

(a) Find all the roots of the polynomials

(i) $f(x) = x^2 + 4x + 3$ [4]

(ii) $f(x) = x^3 - 2x^2 - 3x - 10$ [6]

(b) Find the area under the curve $y = x - x^2$ but above the x-axis. [10]

Question 6

(a) Use the factor theorem to determine whether the given linear expression is a factor of the accompanying polynomial

(i) $5x^4 + 5x^2 - 10; \quad x + 1$

(ii) $3x^3 - 4x^2 + x - 20; \quad x + 4$

[12]

(b) Evaluate the integral

(i) $\int \left(\frac{1}{x} + \frac{1}{x^2} + 4x^5 + 5x^6 \right) \mathrm{d}x$

(ii) $\int \frac{\sin x}{1 + \cos x} \mathrm{d}x$

[8]

Question 7

9a) Find the second derivatives for:

(a) $y = x^5 + 2x^2 + 1$

(b) $y = \sec(3x + 2)$

[10]

(b) Evaluate $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for

$$y = \frac{2x}{4 - x^2}.$$

[10]