UNIVERSITY OF SWAZILAND

FINAL EXAMINATION, MAY 2011

B.A.S.S. I /B.Comm I, D.COM I (IDE)

TITLE OF PAPER	:	CALCULUS FOR BUSINESS AND SOCIAL SCIENC
COURSE NUMBER	:	MS 102 AND IDE MS102
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) Evaluate the following limits

(i) $\lim_{x \to 0} \frac{x^2 + x}{x}$ [3 marks]

(ii) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ [4 marks]

(iii) $\lim_{x \to -1} \frac{x^2 + x}{x - 1}$ [3 marks]

(b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions

(i) $f(x) = \sqrt{x} + 2$ [6 marks]

(ii) $f(x) = x^2 + 1$ [4 marks]

Question 2

Find the derivatives of the following functions

(a) $y = e^{2x} \cos(7x^2 + 1)$ [5 marks]

(b) $y = (3x^3 + 4x^2 + 1)^8$ [5 marks]

(c) $y = \ln \sqrt{\frac{2x + 1}{2x^2 + 1}}$ [5 marks]

(d) $y = 4^{x \cos x}$ [5 marks]

Question 3

A computer firm is marketing a new computer model. It determines that in order to sell x computers, the price per computer must be $p = 280 - 0.4x$. It also determines that the total cost of producing x computers is given by $C(x) = 5000 + 0.6x^2$.

(a) Find the total Revenue function $R(x)$. [3 marks]

(b) Find the total Profit function $P(x)$. [3 marks]

(c) Find the marginal cost. [2 marks]

(d) How many units must the company produce and sell in order to maximize profit. What is the maximum profit and total revenue at this production level. [9 marks]

(e) What price per unit must be charged in order to make this maximum profit. [3 marks]

Question 4

(a) Given the function $f(x) = x^3 - 3x + 3$, find

(i) the $y-$intercept. [1 marks]

(ii) Stationary points. [2 marks]

(iii) Intervals of increase and decrease. [3 marks]

(iv) Relative extrema. [2 marks]

(v) Inflection points. [2 marks]

(vi) Intervals of concavity. [2 marks]

(b) Use all the information obtained in (a) to sketch the graph of the function. [4 marks]

(c) Find the points of discontinuity (if any) of the following function

$$f(x) = \frac{x+1}{x^2 - 4x + 3}$$

[4 marks]

Question 5

Find the following integrals

(a) $\int \left(e^{3x} + 2x^2 + \sin(2x) + \frac{1}{x} + \frac{1}{\sqrt{x}}\right) dx.$ [5 marks]

(b) $\int x\sqrt{x+1}\,dx.$ [5 marks]

(c) $\int \frac{1}{x^2 - 1} dx.$ [5 marks]

(d) $\int x \ln x dx.$ [5 marks]

Question 6

(a) Find the area of the region enclosed by the parabolas

$$y = x^2 \quad \text{and} \quad y = 32 - x^2.$$

[8 marks]

(b) The marginal cost of manufacturing x units of a certain product is

$$C'(x) = 4x^3 - 2x + 1$$

and the fixed cost is $E100$, find

(i) the cost function $C(x)$ [3 marks]

(ii) the increase in total cost, if the the production is increased from 10 to 15 units. [4 marks]

(c) Find the producers surplus at a price level of $E30$ for the supply equation

$$p = S(x) = x + x^2.$$

[5 marks]

Question 7

(a) Given that the function

$$f(x) = 2x^2 + 4x + a$$

passes through the point $(1, 3)$

(i) Find the value of a. [4 marks]

(ii) Find the equation of the tangent to the curve at $x = -2$. [6 marks]

(b) A cattle owner has 800 meters of fence which he wishes to use to make a rectangular holding region in which his cattle will graze. If the region will border an existing fence, calculate the largest area that can be enclosed and find the dimensions of the region. [10 marks]