UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2011

Dip.Comm II, IDE-Dip.Comm III, (B.Ed II)

TITLE OF PAPER   :   QUANTITATIVE TECHNIQUES

COURSE NUMBER   :   MS 202

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF

                                 SEVEN QUESTIONS.

                         2.   ANSWER ANY FIVE QUESTIONS.

                         3.   NON PROGRAMMABLE

                                   CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

1. (a) Minimize the function

$$f(x,y) = xy - 2x + 2$$

subject to $y = x$ using the method of Lagrange multipliers.   [10 marks]

(b) Compute the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ -3 & 0 & 2 \\ -2 & -1 & 0 \end{bmatrix}$$

and use it to solve the linear system

$$\begin{array}{rcrcrcr} 2x & & & - & z & = & 1 \\ -3x & & & + & 2z & = & 0 \\ -2x & - & y & & & = & -2 \end{array}$$

for x, y and z.   [10 marks]

2. (a) Find and classify all critical points of the function
$$f(x,y) = xy - \frac{1}{3}x^3 - \frac{1}{3}y^3.$$ [10 marks]

(b) An economy is based on three industries: agriculture, tourism and transportation.

Each $E1$ in agriculture requires $50c$ in tourism, $20c$ in transportation, and $E1$ in transportation. Each $E1$ in tourism takes $80c$ in tourism and $40c$ in transportation, while each $E1$ in transportation uses $25c$ in agriculture and $10c$ in tourism.

Find the production schedule for the economy if demand is for $E100$ agriculture, $E500$ tourism, and $E700$ transportation.   [10 marks]

3. Michigan Polar Products makes downhill and crosscountry skis. A pair of downhill skis requires 2 man-hours for cutting, 1 man-hour for shaping and 3 man-hours for finishing while a pair of crosscountry skis requires 2 man-hours for cutting, 2 man-hours for shaping and 1 man-hour for finishing. Each day the company has available 140 man-hours for cutting, 120 man-hours for shaping and 150 man-hours for finishing. Formulate the problem as a Linear Programming problem and use the **graphical method** to determine the number of pairs of each type of ski that the company should manufacture each day in order to maximize profit if a pair of downhill skis yields a profit of $10 and a pair of cross-country skis yields a profit of $8? [20 marks]

4. (a) Solve the following linear programming problem by maximizing the dual.

$$\text{Minimize } C = 5x_1 + 9x_2,$$
$$\text{Subject to } x_1 + 5x_2 \geq 2,$$
$$x_1 + x_2 \geq 1$$
$$x_1, x_2 \geq 0.$$

[12 marks]

(b) A company has three warehouses, P, Q and R, which are supplied by three suppliers A, B and C. The table below shows the cost C_{ij} of sending one case of goods from the supplier to the warehouse, in appropriate units. Also shown in the table are the number of cases available at each supplier and the number of cases required at each warehouse. The total number of cases available is equal to the number of cases required. Starting with the Northwest corner solution and using the Stepping Stone Method, determine the transportation pattern that minimizes the total cost.

To From	P	Q	R	Supply
A	5	7	8	70
B	4	4	6	30
C	6	7	7	50
Demand	65	42	43	

[8 marks]

5. (a) A company has 4 machines for assignment to 4 tasks. Any machine can be assigned to any task, and each task requires processing by one machine. The time required to set up each machine for the processing of each task is given in the table below

	TIME(HOURS)			
Machines	Task 1	Task 2	Task 3	Task 4
Machine 1	13	4	7	6
Machine 2	1	11	5	4
Machine 3	6	7	2	8
Machine 4	1	3	5	9

The company wants to minimize the total setup time needed for the processing of all four tasks. Use the Hungarian method to find the allocation that yields the minimum total setup time. [10 marks]

(b) A computer games internet retailer has four favoured customers who each want a copy of the latest *FIFA 2010* computer game. The retailer has one copy available to it at each of two wholesalers in SA, and can get two further copies, one each from each of the two wholesalers in the UK. The costs of each possible allocation of copies (i.e wholesalers) to the customers are

Costs	Customer 1	Customer 2	Customer 3	Customer 4
Wholesaler 1	1	3	6	2
Wholesaler 2	5	2	3	4
Wholesaler 3	9	13	10	8
Wholesaler 4	7	12	8	5

Use the Hungarian Algorithm, find an optimal assignment of wholesalers to customers. [10 marks]

QUESTION 6

6. (a) A debt of E1200 is to be paid off by payments of E500 in 45 days, E300 in 100 days and a final payment of E436.92. Interest is at 11% and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made? [7 marks]

   (b) Sydney wishes to purchase a modest ocean going boat in 5 years time. He figures that he will need E170 000 then. What sum must he invest the end of each quarter in a fund paying 12% compounded quarterly in order to accumulate the price of the boat? [7 marks]

   (c) How much should you deposit in an account paying 6% compounded semi-annually in order to be able to withdraw E1000 every 6 months for the next 3 years? [6 marks]

QUESTION 7

7. Sidlamafa Bottle store carries two competing brands of cheap traditional beer, one from Mbhuleni and the other from Ngwane Park. Both brands of beer can be obtained at a cost of E2 from the store's supplier. Suppose that the Mbhuleni beer is sold at a price of p Emalangeni per bottle and the Ngwane Park beer is sold for q Emalangeni per bottle. The daily demand functions are

$$p = \frac{36}{11} - \frac{7}{110}x - \frac{2}{55}y \quad \text{and} \quad q = \frac{34}{11} - \frac{3}{55}x - \frac{1}{22}y$$

where x is the quantity of the Mbhuleni beer sold and y is the quantity of the Ngwane Park beer sold.

   (a) How many of each type of beer should be sold in order to maximize the total profit? [12 marks]

   (b) Determine the price for each beer that would maximize the total profit from the combined sales. [4 marks]

   (c) What is the maximum profit? [4 marks]