

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2011

Dip.Comm II, IDE-Dip.Comm III

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find and classify all critical points of the function

$$f(x, y) = xy - \frac{1}{3}x^3 - 3x^2 + \frac{1}{4}y^2 + xy + 13x - y + 2. \quad [10 \text{ marks}]$$

- (b) Solve the linear system

$$2x + 5y + 8z = 2$$

$$x + 2y + 3z = 4$$

$$3x + 4y + 4z = 1$$

using Gaussian elimination.

[10 marks]

QUESTION 2

2. (a) Maximize the function

$$g(x, y) = 1 - xy$$

subject to

$$y - x = 2$$

subject to using the method of Lagrange multipliers.

[8 marks]

- (b) An economy has a steel plant, coal mine and transportation.

Each $E1$ worth of steel requires $50c$ from steel plant, $30c$ from coal mine, and $10c$ from transportation. Each $E1$ worth of coal takes $10c$ from steel plant and $20c$ from coal mine and $30c$ from transportation, while each $E1$ worth of transportation uses $10c$ from steel, $40c$ from coal mine, and $5c$ from transportation.

Find the production schedule for the economy if demand is $E2$ million for steel, $E1.5$ million for coal, and $E500,000$ for transportation. [12 marks]

QUESTION 3

3. An electronics company has two machines on its production line.

Machine 1 costs E3 per minute to operate, and it can produce 2 CDs and 1 tape each minute. Machine 2 is older. It costs E2 per minute to operate, and it can produce 1 CD and 1 tape each minute.

The company has orders totaling 6 CDs and 4 tapes.

- (a) The problem is to determine the time in minutes needed to run each machine in order to minimize its costs and still meet its orders. Formulate this as a linear programming problem. [7 marks]
- (b) Solve this linear programming problem by maximizing the dual. [13 marks]

QUESTION 4

4. (a) Solve the linear programming problem

$$\begin{aligned} &\text{Maximize } f(x_1, x_2) = 5x_1 + 10x_2 \\ &\text{subject to } x_1 + 4x_2 \leq 28, \\ &\quad 3x_1 + 2x_2 \leq 24, \\ &\quad x \geq 0, y \geq 0 \end{aligned}$$

using the graphical method. [8 marks]

(b) A company sells x bolts and y nuts each week. Let p and q denote the prices per unit respectively. The demand equations for the company are

$$\begin{aligned} p &= x + 2, \\ q &= y + 3, \end{aligned}$$

and the company's cost function is

$$C(x, y) = 2x^2 + 2y^2 - xy - 4$$

- i. Determine quantities x and y required to maximize weekly profit. Compute the maximum profit. [8 marks]
- ii. Verify that this profit is indeed maximum. [4 marks]

QUESTION 5

5. (a) Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

		Job				
		1	2	3	4	5
Person	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

[10 marks]

- (b) A computer games internet retailer has four favoured customers who each want a copy of the latest *FIFA 2010* computer game. The retailer has one copy available to it at each of two wholesalers in SA, and can get two further copies, one each from each of the two wholesalers in the UK. The costs of each possible allocation of copies (i.e wholesalers) to the customers are

Costs	Customer 1	Customer 2	Customer 3	Customer 4
Wholesaler 1	1	3	6	2
Wholesaler 2	5	2	3	4
Wholesaler 3	9	13	10	8
Wholesaler 4	7	12	8	5

Use the Hungarian Algorithm, find an optimal assignment of wholesalers to customers.

[10 marks]

QUESTION 6

6. (a) A debt of E1200 is to be paid off by payments of E500 in 45 days, E300 in 100 days and a final payment of E436.92. Interest is at 11% and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made? [7 marks]
- (b) Sydney wishes to purchase a modest ocean going boat in 5 years time. He figures that he will need E170 000 then. What sum must he invest the end of each quarter in a fund paying 12% compounded quarterly in order to accumulate the price of the boat? [7 marks]
- (c) How much should you deposit in an account paying 6% compounded semi-annually in order to be able to withdraw E1000 every 6 months for the next 3 years? [6 marks]

QUESTION 7

7. A lumber company ships pine flooring from its three mills, A_1 , A_2 and A_3 , to three building suppliers, B_1 , B_2 and B_3 . The table below shows the demand, availabilities and unit costs of transportation. Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost. [20 marks]

	B_1	B_2	B_3	Availability
A_1	3	3	2	25
A_2	4	2	3	40
A_3	3	4	3	31
Demand	30	30	36	