University of Swaziland

Final Examination, May 2012

BASS I

Title of Paper : Quantitative Techniques II

   Course Number : MS012

   Time Allowed : Three (3) hours

   Instructions :

1. This paper consists of SEVEN questions.

2. Each question is worth 20%.

3. Answer ANY FIVE questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate the following limits

(i) $\quad \lim\limits_{x \to 0^-} \dfrac{1}{x^2}$ [2]

(ii) $\quad \lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ [5]

(iii) $\quad \lim\limits_{x \to \infty} \dfrac{2x^3 + x^2 + 1}{x^2 + 3}$ [5]

(b) Show that

$$\lim\limits_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} = \frac{1}{2}.$$ [8]

Question 2

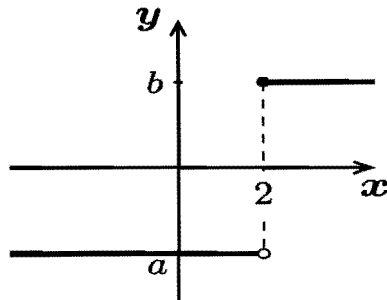(a) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x) = \sqrt{2x + 1}.$$ [10]

(b) From the graph of $f(x)$ shown below, find

(i) $\quad \lim\limits_{x \to 2^-} f(x)$ [3]

(ii) $\quad \lim\limits_{x \to 2^+} f(x)$ [3]

(iii) $\quad \lim\limits_{x \to 2} f(x)$, and explain. [4]

Question 3

(a) Find $f'(x)$ for each of the following functions

   (i) $f(x) = 2x^5 + x^2 + \dfrac{1}{x} + 1$ [2]

   (ii) $f(x) = \left(x^3 + 4x + 3\right)^8$ [4]

   (iii) $f(x) = e^x \sin x + \cos 3x$ [4]

   (iv) $f(x) = \dfrac{x+1}{x+2}$ [4]

(b) If $y = \dfrac{A}{x} + Bx$, where A and B are constants, show that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = y.$$

                                     [6]

                                     [20]

Question 4

(a) Evaluate the following integrals

   (i) $\displaystyle\int 5\,dx$ [2]

(ii) $\displaystyle\int \left(3x^4 + 2x^2 + 1\right)\mathrm{d}x$ [4]

(iii) $\displaystyle\int \sin(4x + 3)\mathrm{d}x$ [4]

(iv) $\displaystyle\int e^{2x+1}\mathrm{d}x$ [4]

(b) Find the area enclosed by the curve $y = -x^2 + 9$ and the x-axis. [6]

Question 5

(a) For the function $f(x) = x^2 - 4x + 3$, find the

(i) stationary points [4]

(ii) intervals of increase and decrease [6]

(b) Make a rough sketch of the curve

$$f(x) = x^3 - 6x^2 + 9x + 1,$$

by considering the x- and y-intercepts, turning points, and intervals of increase/decrease. [10]

Question 6

(a) Show that the graph of the function

$$f(x) = x^3 + x^2 + 5x + 6$$

is always increasing. [6]

(b) If

$$R(v) = \frac{v^2}{4} + \frac{500}{v},$$

find the value of v for which R is minimum. [8]

(c) What type of stationary point(s) does the curve

$$f(x) = -2x^2 - 2x + 1$$

have?   [6]

Question 7

(a) Define Market Equilibrium.   [3]

(b) Find the

    (i)   Equilibrium price

    (ii) Consumer's surplus

    (iii) Producer's surplus

at the equilibrium price level, given that

$$\begin{aligned} p &= D(x) = 20 - 0.05x \\ p &= S(x) = 2 + 0.0002x^2. \end{aligned}$$

[17]