University of Swaziland

Supplementary Examination, July 2012

BASS I

Title of Paper : Quantitative Techniques II

Course Number : MS012

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

Evaluate the following limits

(i) $\lim_{x\to 2}\left(x^2+3x+1\right)$ [3]

(ii) $\lim_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{x-2}$ [6]

(iii) $\lim_{x\to\infty}\left(e^{-x}+1\right)$ [3]

(iv) $\lim_{x\to\infty}\dfrac{2x^2+x+1}{x+4}$ [5]

(v) $\lim_{x\to 0^-}\dfrac{1}{x}$ [3]

Question 2

(a) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x)=\sqrt{x}.$$ [8]

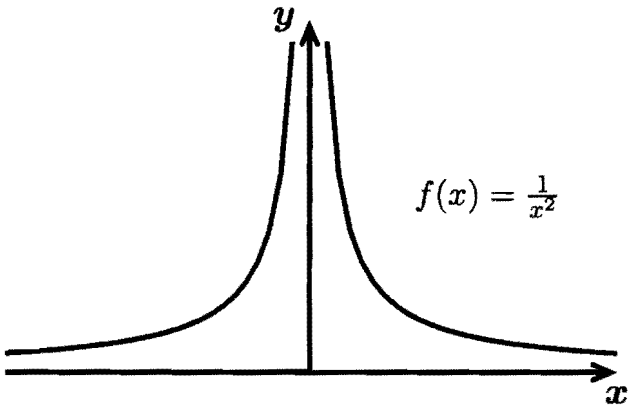
(b) From the graph of $f(x)=\dfrac{1}{x^2}$ shown below, find

(i) $\lim_{x\to 0^-} f(x)$ [3]

(ii) $\lim_{x\to 0^+} f(x)$ [3]

(iii) $\lim_{x\to -\infty} f(x)$, [3]

(iv) the horizontal asymptote of $f(x)$ [3]



Question 3

(a) Find $f'(x)$ for each of the following functions

(i) $f(x) = x^5 + 3x^2 + 2$ [2]

(ii) $f(x) = e^x \sin x$ [4]

(iii) $f(x) = \left(x + \cos x\right)^{10}$ [4]

(iv) $f(x) = \dfrac{x^2 + 1}{x + 2}$ [4]

(b) Find $f''(x)$ for the function $f(x) = \ln\left(x^2 + 1\right)$. [6]

Question 4

(a) Evaluate the following integrals

(i) $\int x^2 \mathrm{d}x$ [2]

(ii) $\int \left(5x^3 + 2x + 7\right)\mathrm{d}x$ []

(iii) $\int \sqrt{2x + 4}\mathrm{d}x$ [4]

(iv) $\displaystyle\int \frac{2x+3}{x^2+3x}\mathrm{d}x$ [3]

(v) $\displaystyle\int \cos 3x \, \mathrm{d}x$ [3]

(b) Find the area enclosed by the curve $y = -x^2 + x + 2$ and the x-axis. [5]

Question 5

(a) What type of stationary point(s) does the curve $f(x) = x^3 - 3x^2 + 3x - 1$ have? [8]

(b) Make a rough sketch of the curve

$$f(x) = x^3 - 3x^2 + 1,$$

by considering the x- and y-intercepts, turning points, and intervals of increase/decrease. [12]

Question 6

(a) A rectangular box, with a lid, is made from a thin metal sheet. Its length is $2x$ cm and its width is x cm. If the box must have a volume of 72 cm^2,

(i) Show that the area A cm^2 of metal used is given by

$$A = 4x^2 + \frac{216}{x},$$ [8]

(ii) Find the value of x so that A is minimum. [6]

(b) The cost $\$C$ of running a boat on a trip is givn by

$$C = 4v^2 + \frac{1000}{v},$$

where v is the average speed in km/h. Find the value of v for which the cost is minimum. [6]

Question 7

(a) Define Market Equilibrium. [3]

(b) Find the

(i) Equilibrium price

(ii) Consumer's surplus

(iii) Producer's surplus

at the equilibrium price level, given that

$$\begin{aligned} p &= D(x) = 20 - 0.05x \\ p &= S(x) = 2 + 0.0002x^2. \end{aligned}$$

[17]