## UNIVERSITY OF SWAZILAND

## FINAL EXAMINATIONS DECEMBER 2011

## B.A.S.S. I /B.COMM I/ D.COMM I



THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

## QUESTION 1

1. (a) Use the long division method to find the quotient and remainder when $x^{4}-3 x^{3}+2 x^{2}-5$ is divided by $x^{2}-x+1 . \quad$ [ 6 marks]
(b) When the polynomial $x^{4}+a x^{3}+11 x^{2}+b x=12$ is divided by $(x+2)$ the remainder is 6 . Given that $(x+4)$ is a factor of the polynomial, find the values of $a$ and $b$.
(c) Find all the real roots of the polynomial

$$
x^{4}+9 x^{3}+21 x^{2}-x-30=0
$$

## QUESTION 2

2. (a) Solve the following equations for $x$
(i) $\log _{2} x+\log _{2}(x-7)=3$

$$
\begin{aligned}
& \text { [5 marks] } \\
& \text { [5 marks] }
\end{aligned}
$$

(ii) $27^{4 x}=9^{x+1}$
(b) Sipho wants to buy a new computer after three years that will cost E5000. How much should he deposit now, at $6 \%$ interest compounded monthly to give the required E5000 in 3 years?
[5 marks]
(c) Find the time required to treble a certain amount compounded continuously at $12 \%$ interest.

## QUESTION 3

3. (a) Express $\sin 840^{\circ}$ as a sine of an acute angle
(b) Prove the following trigonometric identities
i. $\tan \theta+\cot \theta=\sec \theta \csc \theta$
ii. $(1-\cos \theta)(1+\sec \theta)=\sin \theta \tan \theta$ [4 marks]
(c) Solve the trigonometric equation

$$
2 \sin ^{2} x+3 \cos x-3=0
$$

giving all solutions between $0^{\circ}$ and $360^{\circ}$.
(d) Use the sum-difference formulas to find the exact value of $\sin 15^{\circ}$

## QUESTION 4

4. (a) Use the general formula for the $r$ th term to find the coefficient of $x^{6}$ in the binomial expansion of

$$
\left(2+x^{2}\right)^{8}
$$

(b) Use the general formula for the $r$ th term to find the 22nd term in the binomial expansion of

$$
\left(x+\frac{1}{x}\right)^{25}
$$

(c) Use Cramer's rule to solve the following system of equations

$$
\begin{aligned}
x+2 y+z & =1 \\
x-y-z & =0 \\
2 x+y+z & =3
\end{aligned}
$$

## QUESTION 5

5. (a) If the 8 th term of a geometric progression is 243 and the 5 th term is 9 , find the first three terms of the geometric progression.
(b) An auditorium has 40 rows with 30 seats in the first row, 33 in the second row, 36 in the third row, and so forth. How many seats are in the auditorium?
(c) Find the sum of the following series

$$
2+5+8+\ldots \ldots+1001
$$

(d) Convert 5.818181 into an equivalent common fraction

## QUESTION 6

6. (a) Find the equation of $s$ straight line passing through the intersection of $3 x-y=9$ and $x+2 y=-4$, perpendicular to $3=4 y+8 x$
(b) Find the centre and radius of a circle defined by the equation

$$
x^{2}-6 x+y^{2}+10 y+25=0
$$

[6 marks]
(c) Use the synthetic method to divide

$$
x^{5}+3 x^{3}-7 x^{2}-8 x+1 \quad \text { by } \quad x+2
$$

## QUESTION 7

7. (a) Solve the complex quadratic equation

$$
z^{2}-(3-i) z+4=0
$$

and express you answer in the form $x+i y$
[7 marks]
(b) Evaluate $\frac{(1+i)(2+3 i)}{1-i}$ and write the solution in the form $a+b i$ marks]
(c) Prove by mathematical induction that the following formula

$$
5 \cdot 6+5 \cdot 6^{2}+5 \cdot 6^{3}+\cdots+5 \cdot 6^{n}=6\left(6^{n}-1\right)
$$

is valid for all positive integers.
[9 marks]

## END OF EXAMINATION

## Useful Formulas

1. $\sin ^{2} \theta+\cos ^{2} \theta=1$
2. $\sin (A+B)=\sin A \cos B+\cos A \sin B$
3. $\sin (A-B)=\sin A \cos B-\cos A \sin B$
4. $\cos (A+B)=\cos A \cos B-\sin A \sin B$
5. $\cos (A-B)=\cos A \cos B+\sin A \sin B$
6. $2 \cos A \cos B=\cos (A+B)+\cos (A-B)$
7. $\sin 2 A=2 \sin A \cos A$
8. $\cos 2 A=\cos ^{2} A-\sin ^{2} A$
$9 . \cos 2 A=2 \cos ^{2} A-1$
9. $\cos 2 A=1-2 \sin ^{2} A$

| Degrees | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan \theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |  |

