

**UNIVERSITY OF SWAZILAND**

**SUPPLEMENTARY EXAMINATION, JULY 2012**

**B.A.S.S. I /B.Comm I, D.COM I (IDE)**

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| <u>TITLE OF PAPER</u>       | : | CALCULUS FOR BUSINESS AND SOCIAL SCIENCE  |
| <u>COURSE NUMBER</u>        | : | MS 101 AND IDE MS101  |
| <u>TIME ALLOWED</u>         | : | THREE (3) HOURS   |
| <u>INSTRUCTIONS</u>         | : | 1. THIS PAPER CONSISTS OF<br><u>SEVEN</u> QUESTIONS.<br>2. ANSWER ANY <u>FIVE</u> QUESTIONS |
| <u>SPECIAL REQUIREMENTS</u> | : | NONE  |

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL  
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

### Question 1

- (a) Use the long division method to find the quotient and remainder when  $P(x) = 20x^3 + 21x + 18x^2 + 40$  is divided by  $D(x) = 5x + 7$  [7]
- (b) Find all the real roots of the polynomial  $x^4 - x^3 - 19x^2 + 49x - 30$ . [8]
- (c) The expression  $ax^3 + bx + 2$  has  $x + 2$  as a factor. When the expression is divided by  $x - 1$  the remainder is 4. Find the values of  $a$  and  $b$ . [5]

### Question 2

- (a) Calculate  $(A - B)C^T$  if the matrices  $A$ ,  $B$  and  $C$  are given by

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 4 \\ 6 & 3 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 \\ -2 & -4 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & 0 \\ 3 & 2 \end{bmatrix}.$$

[7]

- (b) Use Cramer's rule to solve the following system of equations

$$2x - y + 2z = 2$$

$$x + 10y - 3z = 5$$

$$-x + y + z = -3.$$

[13]

### Question 3

- (a) Write the first four terms of the binomial expansion  $\frac{1}{\sqrt{1+2x}}$ . [5]
- (b) Using the result in 3(a), find the value of  $\frac{1}{\sqrt{1.02}}$  correct to five significant figures. [4]
- (c) Find the term that involves  $x^8$  in the expansion of  $(x^2 - \frac{1}{x})^7$ . [6]
- (d) Expand  $(x + 2y)^4$  using the Binomial theorem. [5]

### Question 4

- (a) Solve for  $x$  in each of the following equations
- (i)  $\log_x \frac{1}{32} = 5$ . [5]
- (ii)  $\log_3(x + 12) - \log_3(x - 3) = \log_3 6$ . [5]
- (b) Suppose that you deposit E8000 on your 30th birthday. What rate  $r$  compounded semi-annually must your deposit earn in order to grow to 1 million Emalangeni by your 70th birthday. [5]
- (c) How much money will grow to E1200 in 5 years at 12% compounded continuously. [5]

### Question 5

- (a) Find the first two terms of the geometric progression whose 3rd term is  $\frac{25}{4}$  and 7th term is  $\frac{4}{25}$ . [5]

(b) The sum of  $n$  terms of the series  $-2, 2, 6, \dots$  is 160. Find  $n$ . [5]

(c) Find the sum of the infinite geometric progression  $\frac{3}{2}, 1, \frac{2}{3}, \dots$  [5]

(d) Convert the repeating decimal  $0.52181818\dots$  into an equivalent common fraction. [5]

### Question 6

(a) Solve the following trigonometric equation

$$\sin x - 2 \sin x \cos x = 0$$

giving all solutions between  $0^\circ$  and  $360^\circ$ . [5]

(b) Prove that

$$\frac{\csc \theta}{\csc \theta - \sin \theta} = \sec^2 \theta.$$

[5]

(c) Using mathematical induction prove that

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n \times (n+1)} = \frac{n}{n+1}$$

for all positive integers  $n$ . [10]

### Question 7

(a) Find the equations of a straight line passing through the intersection of  $3x - y = 9$  and  $x + 2y = -4$  perpendicular to  $3 = 4y + 8x$ . [5]

(b) Find the centre and radius of the circle

$$7x^2 + 7y^2 + 14x - 56y - 25 = 0.$$

[6]

(c) Given the points  $A = (6, 0)$  and  $B = (8, 6)$ . Find the equation of a circle with centre  $A$  and passing through the point  $B$ . [5]

(d) Write the complex number  $3(\cos 135 + i \sin 135)$  in cartesian form  $x + iy$ . [4]

END OF EXAMINATION

### Useful Formulas

1.  $\sin^2 \theta + \cos^2 \theta = 1$
2.  $\sin(A + B) = \sin A \cos B + \cos A \sin B$
3.  $\sin(A - B) = \sin A \cos B - \cos A \sin B$
4.  $\cos(A + B) = \cos A \cos B - \sin A \sin B$
5.  $\cos(A - B) = \cos A \cos B + \sin A \sin B$
6.  $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$
7.  $\sin 2A = 2 \sin A \cos A$
8.  $\cos 2A = \cos^2 A - \sin^2 A$
9.  $\cos 2A = 2 \cos^2 A - 1$
10.  $\cos 2A = 1 - 2 \sin^2 A$

| Degrees       | $0^\circ$ | $30^\circ$           | $45^\circ$           | $60^\circ$           | $90^\circ$ |
|---------------|-----------|----------------------|----------------------|----------------------|------------|
| $\sin \theta$ | 0         | $\frac{1}{2}$        | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1          |
| $\cos \theta$ | 1         | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$        | 0          |
| $\tan \theta$ | 0         | $\frac{1}{\sqrt{3}}$ | 1                    | $\sqrt{3}$           |            |