

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2011/2012

TITLE OF PAPER : CALCULUS FOR BUSSINESS STUDIES

COURSE NUMBER : MS 102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 3

3. Consider the function $f(x) = -2x^3 + 6x^2 - 3$.

(a) Find each of the following.

i. Stationary points. [4 marks]

ii. Inflection points. [2 marks]

iii. Intervals of increase and decrease. [4 marks]

iv. Intervals of concavity. [6 marks]

(b) Use all the information obtained in part 3(a) above to sketch the curve $y = f(x)$. [4 marks]

QUESTION 4

4. (a) A company produces x items at a cost of $C(x) = x^3 - 6x^2 + 15x$. Given that the revenue generated after selling these items is $R(x) = 6x$ after selling these x items, determine a production level that maximizes profit?

[5 marks]

(b) Suppose $C(x) = x^3 - 20x^2 + 20000x$ is the cost of producing x items for a ceratain company. Find a production level that will minimize the average cost of making x items.

[5 marks]

(c) A company manufactures x carts per month. If the monthly cost and price-demand functions are given by

$$C(x) = \frac{x^2}{100} + \frac{x}{2} + 8, \quad p(x) = -\frac{x}{200} + 1$$

Find the following, for each month.

i. Revenue function. [2 marks]

ii. Find the marginal average cost when $x = 30$. Interpret your results. [4 marks]

iii. Find the marginal profit when $x = 30$. Interpret your results. [4 marks]

QUESTION 5

5. Evaluate the following integrals.

(a) $\int \frac{x dx}{x^2 - 3x + 2}$ [5 marks]

(b) $\int \frac{dx}{x^{\frac{2}{3}}(1 + \sqrt[3]{x})}$ [5 marks]

(c) $\int \frac{5}{2} \sec x \tan x dx$ [5 marks]

(d) $\int x \ln x dx$ [5 marks]

QUESTION 6

6. (a) Find the area of the regions enclosed by the following lines and curves.

i. $y = x^2 - 4$ and $y = -x^2 - 2x$. [5 marks]

ii. $y = x^3$ and $y = x^2$. [5 marks]

iii. $y = 2 \sin x$ and $y = \sin 2x$, $0 \leq x \leq \pi$. [5 marks]

(b) Evaluate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (8 \cos x - \sec^2 x) dx$. [5 marks]

QUESTION 7

7. (a) The monthly marginal cost of producing x calculators is given by

$$C'(x) = 12 + \frac{500}{x+1},$$

where $C(x)$ is total cost in Emalangeni.

i. If fixed costs are E1800 per month, find the cost function. [5 marks]

ii. What is the average cost if 900 calculators are produced each month? [5 marks]

(b) A company has marginal revenue marginal cost given by

$$R'(x) = \frac{2400}{(10+x)^2} \text{ and } C'(x) = 0.02x + 5$$

If the company increases production from $x = 30$ to $x = 40$ units, compute the change in total profits. [5 marks]

(c) Given the demand function $D(x) = 70 - 0.05x^2$ and the supply function $S(x) = 22 + 0.1x$, find the the producers surplus. [5 marks]