UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2012

B.A.S.S. I /B.Comm I, D.COM I (IDE)

TITLE OF PAPER   :   CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER   :   MS 102 AND IDE MS102

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF
                                     SEVEN QUESTIONS.
                     2.   ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Question 1

(a) The function $f(x)$ is defined by

$$f(x) = \begin{cases} \frac{x^2-16}{x+4}, & \text{if } x \neq -4; \\ 8, & \text{if } x = -4. \end{cases}$$

Is this function continuous at $x = -4$? Explain. [3]

(b) Evaluate the following limits

(i) $\displaystyle\lim_{x\to\infty} \frac{5x^6 - x^4 + 9}{4x^7 + 1}$ [4]

(ii) $\displaystyle\lim_{x\to 0} \frac{x^3 + x^2 + x}{x}$ [3]

(iii) $\displaystyle\lim_{x\to 9} \frac{\sqrt{x}(\sqrt{x} - 3)}{x - 9}$ [5]

(c) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following function $f(x) = \dfrac{1}{x}$ [5]

Question 2

Find the derivatives of the following functions

(a) $y = \dfrac{1}{\sqrt{2x + 7x^2 + e^x}}$ [5]

(b) $y = \dfrac{\cos(3x)}{\ln x}$ [5]

(c) $y = (x^2 + 1)e^{x^2+1}$ [5]

(d) $y = 2^{x^2}$ [5]

Question 3

(a) Given the function $y = 2x^3 + 3x^2 - 12x + 7$, find the

   (i) local maximum.   [2]

   (ii) local minimum.   [2]

   (iii) point of inflection.   [2]

(b) Find the intervals where the curve is

   (i) increasing.   [2]

   (ii) decreasing.   [2]

   (iii) concave up.   [3]

   (iv) concave down.   [3]

(c) Use all the information obtained in (a) and (b) to sketch the graph of the function.   [4]

Question 4

(a) Find the first four derivatives of the following function

$$y = \ln(x^5 e^{x^2}).$$

[4]

(b) Find the equation of the tangent to the curve

$$y = x^4 - 8x^2$$

at $x = 1$.   [4]

(c) A printer needs to print a document that contains 50 square centimetres of printed material with a 2cm margin on the right and left hand sides of

the paper and a 4cm margin on the top and bottom of the paper. What is the minimum size of paper that can fulfill these requirements.   [12]

Question 5

(a) Find the area bounded by the graphs

$$y = x^2 \text{ and } y = x + 2.$$

[5]

(b) Given the demand function $D(x)$ and the supply function $S(x)$

$$p = D(x) = 300 - 6x - x^2, \qquad p = S(x) = x^2 + 4x$$

find the

(i) equilibrium price   [3]

(ii) consumer surplus   [6]

(ii) producer surplus   [6]

Question 6

Find the following integrals

(a) $\int \dfrac{x + 4}{x^2 + 5x - 6} dx.$ [5]

(b) $\displaystyle \int (8 + e^{2x} + \sqrt{x} + \cos(7x))dx.$ [5]

(c) $\displaystyle \int x(\sqrt{x} + 1)dx.$ [5]

(d) $\int x^2 e^x dx.$ [5]

Question 7

(a) Find the equation of the curve that passes through $(1,3)$ if its slope is given by

$$\frac{dy}{dx} = 12x^2 - 12x$$

for each value of x. [4]

(b) The rate of growth of the population $N(t)$ of a new city t years after its incorporation is estimated to be

$$\frac{dN}{dt} = 400 + 600\sqrt{t}.$$

If the population was 5000 at the time of incorporation, find the population 9 years later. [6]

(c) The marginal revenue and marginal cost of producing x units are given by

$$R'(x) = -.6x + 76$$

$$C'(x) = 20.$$

The fixed cost is $E638$, find the profit obtained when 20 items are sold. [10]