UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2011/2012

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :
1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Solve the linear system

$$\begin{aligned} 3x + y - 2z &= -7 \\ 2x + 2y + z &= 9 \\ -x - y + 3z &= 6 \end{aligned}$$

using Cramer's rule. [10 marks]

(b) Find and classify all critical points of the function $f(x, y) = x^3 - y^3 - 3x + 3y$. [10 marks]

QUESTION 2

2. (a) The economic cooperation between 3 industries A, B and C in a year is shown in table (1).

Input \ Output	A	B	C	External demand	Total output
A	25	15	10	50	100
B	15	10	5	10	40
C	10	20	5	15	50

Table 1: Transaction table for economy with 3 industries

If forecast external demand in 3 years is $[60, 70, 90]^T$, what should the total output be? [12 marks]

(b) Consider the following problem:

$$\text{Maximize } f(x, y) = xy + y$$

subject to

$$x + y = 3$$

i. Solve this problem using the method of Lagrange multipliers. [6 marks]

ii. Determine the maximum value of the function $f(x, y)$. [2 marks]

QUESTION 3

3. A company manufactures stools and tables. Each stool requires 1 hour of carpentry, 1 hour of painting and 2 hours of finishing. Similarly, a table needs 2 hours of carpentry, 1 hour of painting and 1 hour of finishing. During each production period, the carpentry, painting and finishing departments can only work for up to 10 hours, 7 hours and 12 hours respectively. The company makes E40 profit per stool and E30 profit per table.

   (a) The problem is to determine the number of stools and tables that should be made in order to maximize profits. Formulate this as a linear programming problem. [8 marks]

   (b) Solve linear programming problem by the graphical method. [12 marks]

QUESTION 4

4. Two dietary drinks are used to supply protein and carbohydrates. The first drink provides 1 unit of protein and 3 units of carbohydrates in each litre. The second drink supplies 2 units of protein and 2 units of carbohydrates in each litre. An athlete requires 3 units of protein and 5 units of carbohydrates. The first drink costs E2 per litre and the second costs E3 per litre.

   (a) The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem. [8 marks]

   (b) Solve linear programming problem by maximizing the dual. [12 marks]

QUESTION 5

5. An electricity company ships coal from 3 collieries, X_1, X_2 and X_3, to its 3 power stations, Y_1, Y_2 and Y_3. Table (2) shows the demand, availabilities and unit costs of transportation.

	Y_1	Y_2	Y_3	Availability
X_1	3	3	2	50
X_2	4	2	3	80
X_3	3	4	3	62
Demand	60	60	72	

Table 2: Demand, supply and unit cost values

Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost. [20 marks]

QUESTION 6

6. (a) A company wishes to assign its employees $1, 2, 3, 4, 5$ to 5 different training courses based on their skills. The assignment costs are given as follows:

Cost	A	B	C	D	E
1	14	7	3	7	27
2	20	7	12	6	30
3	10	3	4	5	21
4	8	12	7	12	21
5	13	25	24	26	8

Determine the optimal assignment schedule that minimizes the total cost.

[10 marks]

(b) A company has 4 employees $1, 2, 3, 4$ to assign to 4 projects A, B, C, D based on the following scores:

Score	A	B	C	D
1	20	16	22	18
2	25	28	15	21
3	27	20	23	26
4	24	22	23	22

Determine the optimal assignment schedule that maximizes the total score.

[10 marks]

QUESTION 7

7. (a) A loan of $E1300$ is due in 20 weeks with interest charged at 15% per annum. The debtor makes a first payment of $E520$ in 5 weeks, followed by a payment of $E480$ in 13 weeks. Find the balance payable on due date under the Merchant's rule. [6 marks]

   (b) A notebook computer can be purchased using only one of two options. The first option is to pay $E2200$ cash. The second option requires a down payment of $E700$ followed by payments of $E100$ every month for 18 months. If interest charged is at rate 11% compounded monthly, are the two options equivalent? [8 marks]

   (c) What sum of money should be set aside to provide an income of $E850$ every 3 months for the next 5 years if the money earns interest at rate 7.5% compounded quarterly? [6 marks]