UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2011/2012

TITLE OF PAPER : QUANTITATIVE TECHNIQUES

COURSE NUMBER : MS 202

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :
1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find and classify all critical points of the function $f(x, y) = x^3 + y^3 - 3x - 3y$. [10 marks]

   (b) Solve the linear system

$$\begin{array}{rcrcrcr} 3x & + & y & - & 2z & = & -7 \\ -x & - & y & + & 3z & = & 6 \\ 2x & + & 2y & + & z & = & 9 \end{array}$$

   using Gauss-Jordan elimination. [10 marks]

QUESTION 2

2. (a) Consider the following problem:

$$\text{Minimize } f(x, y) = xy - 3y$$

   subject to

$$x - y = 1$$

   i. Solve this problem using the method of Lagrange multipliers. [6 marks]

   ii. Determine the minimum value of the function $f(x, y)$. [2 marks]

   (b) An economy is based on 3 industries: mining, energy and clothing.

   Each $E1$ in mining requires $50c$ in energy, $20c$ in clothing, and $E1$ in clothing. Each $E1$ in energy takes $80c$ in energy and $40c$ in clothing, while each $E1$ in clothing uses $25c$ in mining and $10c$ in energy.

   Find the production schedule for the economy if demand is for $E10$ million in mining, $E50$ million in energy, and $E70$ million in clothing. [12 marks]

QUESTION 3

3. A company manufactures desks and chairs. Each desk requires 1 hour of carpentry, 1 hour of painting and 2 hours of finishing. Similarly, a chair needs 2 hours of carpentry, 1 hour of painting and 1 hour of finishing. During each production period, the carpentry, painting and finishing departments can only work for up to 10 hours, 7 hours and 12 hours respectively. The company makes E40 profit per desk and E30 profit per chair.

   (a) The problem is to determine the number of desks and chairs that should be made in order to maximize profits. Formulate this as a linear programming problem. [8 marks]

   (b) Solve linear programming problem by the simplex method. [12 marks]

QUESTION 4

4. Two dietary drinks are used to supply vitamin C and vitamin D. The first drink provides 2 units of vitamin C and 1 unit of vitamin D in each litre. The second drink supplies 7 units of vitamin C and 2 units of vitamin D in each litre. An athlete requires 9 units of vitamin C and 4 units of vitamin D. The first drink costs E3 per litre and the second costs E8 per litre.

   (a) The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem. [8 marks]

   (b) Solve linear programming problem by maximizing the dual. [12 marks]

QUESTION 5

5. A clothing company ships cotton from 3 farms, A, B and C, to its 3 warehouses, X, Y and Z. Table (1) shows the demand, availabilities and unit costs of transportation.

	X	Y	Z	Availability
A	10	12	9	40
B	4	5	7	50
C	11	8	6	60
Demand	70	50	30	

Table 1: Demand, supply and unit cost values

Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost. [20 marks]

QUESTION 6

6. (a) A company wishes to assign 4 of its taxi drivers $1, 2, 3, 4$ to 4 different routes A, B, C, D. The assignment costs are given as follows:

Cost	A	B	C	D
1	90	75	75	80
2	35	85	55	65
3	125	95	90	105
4	45	110	95	115

Determine the optimal assignment schedule that minimizes the total cost. [10 marks]

(b) A company has 4 employees $1, 2, 3, 4$ to assign to 4 projects A, B, C, D based on the following scores:

Score	A	B	C	D
1	80	55	45	45
2	58	35	70	50
3	70	50	80	65
4	90	70	40	80

Determine the optimal assignment schedule that maximizes the total score. [10 marks]

QUESTION 7

7. (a) A loan of $E1500$ is due in 10 months with interest charged at 7.5% per annum. The debtor makes a first payment of $E610$ in 4 months, followed by a payment of $E530$ in 7 months. Find the balance payable on due date under the Merchant's rule. [6 marks]

   (b) A T.V set can be purchased using only one of two options. The first option is to pay $E1300$ cash. The second option requires a down payment of $E500$ followed by payments of $E50$ every month for 24 months. If interest charged is at rate 6% compounded monthly, are the two options equivalent? [8 marks]

   (c) What sum of money should be set aside to provide an income of $E1200$ every month for the next 3 years if the money earns interest at rate 12% compounded monthly? [6 marks]