University of Swaziland

Final Examination, December 2012

BA Social Science I

Title of Paper : Elementary Quant. Techniques I

Course Number : MS011

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

Useful Formulae

Arithmetic Progressions

$$T_n = T_1 + (n-1)d, \qquad S_n = \begin{cases} \frac{n}{2}\left[T_1 + T_n\right] \\ \frac{n}{2}\left[2T_1 + (n-1)d\right] \end{cases}$$

Geometric Progressions

$$T_n = T_1 r^{n-1}, \qquad S_n = \frac{T_1(1-r^n)}{1-r}.$$

Binomial Theorem

$$\left(a+b\right)^n = a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^2 + {}_nC_3 a^{n-3}b^3 + \cdots + b^n.$$

Matrices

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

Logarithms and Exponential Functions

- $\log_b y = x \iff b^x = y.$
- $\log_b AB = \log_b A + \log_b B$
- $\log_b \left(\frac{A}{B}\right) = \log_b A - \log_b B$
- $\log_b A^n = n \log_b A$

Exponential Growth/Decay

	Discontinuous (re-valued once a year)	**Continuous**
Growth	$Q(t) = Q_0(1+r)^t$	$Q(t) = Q_0 e^{rt}$
Decay	$Q(t) = Q_0(1-r)^t$	$Q(t) = Q_0 e^{-rt}$

Question 1

(a) Given the matrices

$$B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 1 & -2 & 1 \\ 3 & 0 & 2 \end{pmatrix}$$

work out (where possible)

i. $2B + C$ ii. $B^T - C$

iii. BC iv. BD

v. DC

[10 marks]

(b) Work out

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix}.$$

[4 marks]

(c) Solve the system

$$\begin{aligned} 2x - 5y &= -26 \\ 6x + 7y &= 10 \end{aligned}$$

using Cramer's rule. [6 marks]

Question 2

(a) Consider the *arithmetic progression*

$$100, 92, 84, 76, \cdots .$$

Find

i. the next 2 terms [2 marks]

ii. the values of T_1 and d [1 mark]

iii. the formula for the n-th term [3 marks]

iv. the 50th term [1 mark]

v. sum of the first 30 terms [4 marks]

(b) A parent sets up a fund for his child by making monthly savings. He deposits E700, E760, and E820 at the end of the first, second and third months, respectively – the amounts increasing by E60 every month. Find

i. the instalment after 1 year [2 marks]

ii. when the intalment will reach E1,600 [3 marks]

iii. the total deposited after 2 years [4 marks]

Question 3

(a) Simplify and leave your answer in terms of positive indices

i. $A^{-3} \times A^7$ [1 mark]

ii. $A^3 \div A^{-7}$ [1 mark]

iii. $\left(A^{-3}\right)^7$ [1 mark]

iv. $\left(\frac{A^{-3}}{A^7}\right)^{-2}$ [2 marks]

v. $\left(\frac{A^{-3}}{A^{-7}}\right)^2$ [2 marks]

(b) Use the binomial theorem to expand

$$(x-2y)^5$$

and simplify term by term. [6 marks]

(c) In the binomial expansion of

$$\left(1+u^2\right)^{20},$$

find

i. the first 4 terms [4 marks]

ii. the 17th term [3 marks]

Question 4

(a) Use long division to work out

$$\frac{x^3 - 2x^2 + 3x - 4}{x + 2}.$$ [8 marks]

(b) Use synthetic division to work out

$$\frac{x^3 + x^2 - 2x - 4}{x + 2}.$$ [4 marks]

(c) Consider the polynoomial

$$P(x) = x^3 + 2x^2 + Ax - 6.$$

where A is a constant.

i. Find the value of A given that $(x + 1)$ is a factor of $P(x)$. [2 marks]

ii. By first dividing by $(x + 1)$, completely factorise $P(x)$ [4 marks]

iii. Hence find the roots of $P(x) = 0$. [2 marks]

Question 5

(a) Find the value of

i. $\log_3 9,000$ [2 marks]

ii. $\log_{10} 0.001$ [2 marks]

(b) Solve for x

i. $3^{x-1} = 81$ [3 marks]

ii. $\log_3(4x - 7) = 2$ [3 marks]

iii. $\log x + \log x = 4$ [3 marks]

(c) A company buys a new vehicle for E450,000 on 01 January 2012. If its value depreciates by 7.8% per year (re-valued once a year), find the

i. value of the vehicle after 5 years; [2 marks]

ii. the date corresponding to the *half-life* of the vehicle. [5 marks]

Question 6

(a) The coordiantes of two cities are $A(-20, 40)$ and $B(30, -60)$, where the coordinates are in kilometres. Find the

i. distance between the 2 cities, correct to the nearest kilometre [3 marks]

ii. equation of the straight line AB [4 marks]

iii. coordinates of the midpoint of AB [2 marks]

iv. equation of the perpendicular bisector of AB [4 marks]

(b) Consider the equation of the straight line

$$\ell : \quad 12x - 4y = 7.$$

Find the

i. gradient of the line ℓ [3 marks]

ii. equation of a straight line parallel to ℓ passing through $(1, 2)$ [4 marks]

Question 7

(a) Consider the *geometric progression*

$$2, 6, 18, \cdots .$$

Find

i. the next 2 terms [2 marks]

ii. the values of T_1 and r [1 mark]

iii. the formula for the n-th term [2 marks]

iv. the 10th term [2 marks]

v. sum of the first 10 terms [3 marks]

(b) Find the value of

$$\sum_{n=0}^{100}(6n+5)$$ [3 marks]

(c) The population of a city grows according to

$$P(t) = 16,000e^{0.02t}$$

where t is the number of years from year 2000. Find

i. the population of the city in year 2000; [2 marks]

ii. the population of the city in 2010; [2 marks]

iii. the date when the population will be double that in 2000. [3 marks]