

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2012/2013

TITLE OF PAPER : ALGEBRA, TRIGONOMETRY AND
ANALYTIC GEOMETRY

COURSE NUMBER : MS 101

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find the values of a and b for which the polynomial $x^3 + ax^2 + bx - 4$ is exactly divisible by $x^2 + x - 2$. [7 marks]
- (b) Use both the remainder theorem and synthetic division to find all the real roots of the equation $3x^4 + 5x^3 - 10x^2 - 20x - 8 = 0$. [13 marks]

QUESTION 2

2. (a) Evaluate $\log_3 33$. [3 marks]
- (b) Solve the following equations for x .
- i. $3^{2x-1} = 37$. [4 marks]
- ii. $\log_2 x + \log_2 8 = 6$. [5 marks]
- iii. $2e^{2x} - 7e^x + 3 = 0$. [4 marks]
- (c) How long will it take for money in an account that is compounded continuously at 5% interest to trebble? [4 marks]

QUESTION 3

3. (a) Express $\cos 725^\circ$ in terms of an acute angle. [3 marks]
- (b) Prove the following trigonometric identities.
- i. $\tan x + \cot x = 2 \operatorname{cosec} 2y$. (NB $\operatorname{cosec} x = \csc x$) [4 marks]
- ii. $\frac{\sin y + \sin 3y}{\cos y + \cos 3y} = \tan 2y$ [9 marks]
- (c) Solve the trigonometric equation

$$\sin x = \sin 2x$$

and give your answers in the range $0^\circ \leq x < 360^\circ$. [4 marks]

QUESTION 4

4. (a) Find

i. the constant term, and

ii. the term involving x^6 ,

in the binomial expansion of $\left(x + \frac{1}{2x}\right)^8$. [5,5 marks]

(b) i. Write down the first 4 terms of the binomial expansion of

$\frac{1}{\sqrt{1-x}}$. [4 marks]

ii. Use your result in part 4(b)i above to estimate $\frac{1}{\sqrt{99}}$ correct to 4 decimal places. [6 marks]

QUESTION 5

5. (a) Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \text{ and } C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

i. Find $(A - B)^T$. [3 marks]

ii. Verify that $A^T - B^T = (A - B)^T$. [4 marks]

iii. Find $A^T C - B^T C$. [3 marks]

(b) Solve the linear system

$$\begin{array}{rcl} x & - & z = 1 \\ 2x + 3y & - & 5z = 0 \\ 7x & + & 2z = -1 \end{array}$$

using Cramer's rule. **Leave your answers as fractions.** [10 marks]

QUESTION 6

6. (a) Given the circle

$$x^2 + 10x + y^2 - 4y = 7$$

find its centre and radius. [6 marks]

- (b) Write down the equation of the circle that is tangent to the y -axis and has centre $(-3, 2)$. [5 marks]

- (c) Find the equation of the line which is parallel to the line $3x - 2y - 4 = 0$ and which passes through the point $(0, 2)$. [4 marks]

- (d) Write down the equation of the line which is perpendicular to the line $3x - 4y = 2$ and which passes through the point $(-1, 1)$. [5 marks]

QUESTION 7

7. (a) The sum of an infinite geometric progression is $\frac{8}{3}$ and the first term is 2. Find the common ratio. [4 marks]

- (b) Use mathematical induction to prove that

$$\frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

for any positive integer n . [6 marks]

- (c) Simplify the complex number

$$\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$$

without using a calculator. [4 marks]

- (d) Solve the equation

$$z^2 = 4\bar{z}$$

and give your answer(s) in the form $z = x + iy$. [6 marks]