

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2013

B.A.S.S. I /B.Comm I, D.COM I (IDE)

- TITLE OF PAPER : ALGEBRA, TRIGONOMETRY AND  
ANALYTIC GEOMETRY
- COURSE NUMBER : MS 101 AND IDE MS101
- TIME ALLOWED : THREE (3) HOURS
- INSTRUCTIONS : 1. THIS PAPER CONSISTS OF  
SEVEN QUESTIONS.  
2. ANSWER ANY FIVE QUESTIONS
- SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL  
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

### Question 1

- (a) Use the long division method to find the quotient and remainder when  $P(x) = x^4 + x^3 - x - 3$  is divided by  $D(x) = x + 3$  [6]
- (b) By using the remainder theorem and synthetic division, find all the real roots of the polynomial  $2x^4 - 2x^3 - 14x^2 + 2x + 12 = 0$ . [8]
- (c) The expression  $ax^3 + bx^2 - 3x + 18$  has  $x - 2$  as a factor. When the expression is divided by  $x - 1$  the remainder is 10. Find the values of  $a$  and  $b$ . [6]

### Question 2

- (a) Solve the following equations
- (i)  $\log_2(x) + \log_2(x + 2) = 3$ . [5]
- (ii)  $3^{2x+1} = 5^{x+1}$ . [5]
- (b) Find the annual interest rate required to treble a certain amount if the interest is compounded monthly for 10 years. [5]
- (c) A student wants to buy a new computer after 4 years that will cost  $E8000$ . If she only has  $E4000$  available to deposit now, what interest rate is required for it to increase to  $E8000$  in 4 years if the interest is compounded continuously. [5]

### Question 3

- (a) Prove the trigonometric identity

$$(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x.$$

[5]

- (b) Solve the trigonometric equation

$$2 \sin^2 x + \sin x - 1 = 0$$

giving all solutions between  $0^\circ$  and  $360^\circ$ .

[8]

- (c) Convert the decimal 1.3712712712.... into a common fraction.

[3]

- (d) Find the sum of the following progression

$$2 + 5 + 8 + 11 + \dots + 1001.$$

[4]

### Question 4

- (a) For the following expansion  $\left(2x^2 - \frac{1}{4x^2}\right)^{22}$ , find the

(i) 8th term.

[5]

(ii) constant term.

[6]

- (b) Write the first four terms of the expansion  $\frac{1}{\sqrt{1+x}}$  and use the expansion to estimate  $\frac{1}{\sqrt{1.03}}$  correct to five significant figures.

[9]

### Question 5

- (a) Use Cramer's rule to solve the following system of equations

$$2x - y + 2z = 2$$

$$x + 10y - 3z = 5$$

$$-x + y + z = -3$$

[13]

- (a) Calculate  $(A - B)C^T$  if the matrices  $A$ ,  $B$  and  $C$  are given by

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 4 \\ 6 & 3 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 \\ -2 & -4 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & 0 \\ 3 & 2 \end{bmatrix}.$$

[7]

### Question 6

- (a) Write the equation of the circle with center  $(2, -7)$  and which passes through the point  $(-2, -4)$ . [3]
- (b) Find the center and radius of the circle given by  $x^2 + y^2 + 3x - 5y - \frac{1}{2} = 0$ . [4]
- (c) Find the equation of a straight line passing through the intersection of  $3x - y = 9$  and  $x + 2y = -4$  and is perpendicular to  $2y + 8x = 3$ . [7]
- (d) Find an equation of the line parallel to the line  $y = -2x + 3$  and which passes through the point  $(2, 5)$ . [6]

### Question 7

(a) Express the following expressions in the complex form

(i)  $(3 - 4i)(-3 - 2i)$ . [2]

(ii)  $\frac{3 - 4i}{2 - 3i}$ . [4]

(iii)  $\frac{\sqrt{2}}{3}(\cos 225^\circ + i \sin 225^\circ)$ . (without using a calculator) [4]

(c) Using mathematical induction prove that

$$1 + 3 + 6 + \dots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}.$$

[10]