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# University of Swaziland



Final Examination, 2012/2013

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**BCom I, Bass I, IDE-DCom I, IDE-BCom I,  
IDE-BEd Com I**

**Title of Paper** : Calculus for Business Studies

**Course Number** : MS102/IDE-MS102

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

**THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS  
BEEN GIVEN BY THE INVIGILATOR.**

**QUESTION 1**

1.1. Evaluate the following limits.

1.1.1.  $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^2 + 3x + 2}$  (4)

1.1.2.  $\lim_{x \rightarrow \infty} \frac{5 + 3x - 4x^2}{4 + 5x^2}$  (4)

1.2. Let

$$f(x) = \begin{cases} ax + 2 & \text{when } x \leq 1, \\ 4x^2 & \text{when } x > 1. \end{cases}$$

Find a value of  $a$  that will make  $f(x)$  continuous at  $x = 1$ . (6)1.3. Use the limit definition of the derivative to find  $f'(x)$  if

$$f(x) = \sqrt{2x + 1}. \quad (6)$$

**QUESTION 2**

Find the derivatives of the following functions.

2.1.  $f(x) = \frac{x^2}{2} - \frac{x}{4} + \frac{1}{2x^2} - \frac{2}{\sqrt{x}}$  (4)

2.2.  $f(x) = x^2 e^{-x^2}$  (5)

2.3.  $f(x) = \frac{1 - \cos x}{1 + \cos x}$  (5)

2.4.  $f(x) = x^x$  (6)

**QUESTION 3**

Find the following integrals.

3.1.  $\int \left( 3x^2 + e^{2x} + \frac{1}{2x} + \cos x \right) dx$  (5)

3.2.  $\int \frac{\ln x}{x} dx$  (5)

3.3.  $\int \frac{2x + 1}{x^2 - 1} dx$  (5)

3.4.  $\int x^2 \cos x dx$  (5)

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**QUESTION 4**

- 4.1. A company manufactures floor mops. The company estimates that if it produces  $x$  mops per month, it can charge a price  $p$  emalangeni given by

$$p(x) = 50 - \frac{x}{500}$$

per mop. The cost (in emalangeni) of producing  $x$  mops per month is given by

$$C(x) = 140000 + 10x.$$

- 4.1.1. Write down the company's profit function. (3)
- 4.1.2. Determine the company's marginal profit when production is 5000 mops per month and interpret the result. (3)
- 4.1.3. How many mops should be produced per month in order to maximize the company's monthly profits? (6)
- 4.2. The marginal cost of producing  $x$  units of a particular product is

$$C'(x) = \frac{x}{25}.$$

- 4.2.1. Estimate the cost of producing the twenty-sixth item. (2)
- 4.2.2. Given that the fixed costs amount to E20, find the total cost of producing 100 items. (4)
- 4.2.3. Find the exact cost of producing the twenty-sixth item. (2)

**QUESTION 5**

- 5.1. Given that the function

$$f(x) = 3x^5 - ax^3 + 3$$

passes through the point  $(-1, 5)$ ,

- 5.1.1. find the value of  $a$ , and then (4)
- 5.1.2. find the equation of the tangent to the curve at  $x = 2$ . (6)
- 5.2. A farmer has 1000 metres of fencing material and wants to fence a rectangular area with one side bordering a straight stream. If the side along the stream needs no fencing, find the dimensions that make the enclosure as large as possible. What is the maximum area? (10)

**QUESTION 6**

Let  $f(x) = 2x^3 - 3x^2 - 12x + 1$ .

- 6.1. Find the  $y$ -intercept for the curve  $y = f(x)$ . (1)
- 6.2. Find the intervals on which  $f$  is increasing and the intervals on which  $f$  is decreasing. (5)
- 6.3. Find the local maximum and local minimum values of  $f$ . (3)
- 6.4. Find the intervals on which  $f$  is concave up and the intervals on which  $f$  is concave down. (5)
- 6.5. Find the inflection points for the curve  $y = f(x)$ . (2)
- 6.6. Sketch a graph of  $f$  using the information in (6.1) to (6.5). (4)

**QUESTION 7**

Suppose the demand function for a commodity is

$$p = D(x) = 100 - 0.001x^2$$

and its supply function is

$$p = S(x) = 40 + 0.1x.$$

- 7.1. Find the equilibrium price and the quantities exchanged at equilibrium. (6)
- 7.2. Find the consumer's surplus. (7)
- 7.3. Find the producer's surplus. (7)

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END OF EXAMINATION PAPER

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