

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2012/2013

TITLE OF PAPER : CALCULUS FOR BUSSINESS STUDIES

COURSE NUMBER : MS 102

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE
CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits.

i. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{2x^2 - x - 3}$. [5 marks]

ii. $\lim_{x \rightarrow 1} \frac{3 - 3x}{x - \sqrt{x}}$. [5 marks]

iii. $\lim_{x \rightarrow -\infty} \frac{x^4 + 2}{5x^4 - 4x^2 - 3}$. [5 marks]

(b) Let a and b be constants, and let

$$f(x) = \begin{cases} a, & \text{if } x = 1 \\ bx^2 - 1, & \text{if } x \neq 1 \end{cases}$$

be a function. Suppose that

- f is continuous at $x = 1$, and
- the slope of the tangent to the curve $y = f(x)$ at $x = -1$ is 2.

Find suitable values for a and b . [5 marks]

QUESTION 2

2. (a) Use the limit definition of the derivative of a function to find $f'(x)$ for each of the following functions.

i. $f(x) = 3x^2$. [5 marks]

ii. $f(x) = 7 - \sqrt{x}$. [5 marks]

(b) Find derivatives of the following functions

i. $f(x) = e^{2x} \tan 3x$ [5 marks]

ii. $f(x) = 5^{\ln 7x}$ [5 marks]

QUESTION 3

3. Let $f(x) = 2x^3 - 4x^2 - 2x + 4$ be a function.

- (a) i. Find all stationary points of f . [3 marks]
 - ii. Determine the intervals of increase and decrease of f . [4 marks]
 - iii. Use the first derivative test to classify all stationary points of f . [3 marks]
 - iv. Find all points of inflection of f . [2 marks]
 - v. Determine the intervals of concavity of f . [4 marks]
- (b) Use all the information obtained in part 3(a) above to sketch the curve $y = f(x)$. [4 marks]

QUESTION 4

4. (a) A company produces x items at a cost of $C(x) = x^3 - 6x^2 + 15x$. Given that the revenue generated after selling these items is $R(x) = 6x$, determine a production level that maximizes profit? [5 marks]
- (b) Suppose $C(x) = x^3 - 20x^2 + 20000x$ is the cost of producing x items for a certain company. Find a production level that will minimize the average cost of making x items. [5 marks]
- (c) A company manufactures x carts per month. If the monthly cost and price-demand functions are given by

$$C(x) = \frac{x^2}{100} + \frac{x}{2} + 10, \quad p(x) = -\frac{x}{200} + 1$$

Find the following, for each month.

- i. Revenue function. [2 marks]
- ii. Find the marginal average cost when $x = 30$. Interpret your results. [4 marks]
- iii. Find the marginal profit when $x = 30$. Interpret your results. [4 marks]

QUESTION 5

5. Evaluate the following integrals.

(a) $\int \left(5x^4 + e^{-2x} - \frac{2}{3x} + \sin x \right) dx$ [5 marks]

(b) $\int 2x(x^2 + 1)^4 dx$ [5 marks]

(c) $\int \frac{1}{x^2 - 4} dx$ [5 marks]

(d) $\int x \sin x dx$ [5 marks]

QUESTION 6

6. (a) Find the area of the region enclosed by the following curve and line.

$$y = x^2 \text{ and } y = -x + 2.$$

[10 marks]

(b) Given the demand function $D(x) = 400 - 30x^2$ and the supply function $S(x) = 10x^2 + 120x$, find the consumer surplus.. [10 marks]

QUESTION 7

7. (a) The marginal cost for a weekly demand of x bottles of shampoo in a pharmacy is given by

$$p'(x) = -\frac{6000}{(3x + 50)^2}.$$

i. Find the price-demand equation if the weekly demand is 150 when the price of the bottle is $E4$. [5 marks]

ii. What is the weekly demand if the price is $E2.50$. [5 marks]

(b) The weekly marginal revenue from the sale of x items is given by

$$R'(x) = \frac{200}{x + 1} + 40 - 0.02x.$$

Find the revenue function.

[10 marks]