# UNIVERSITY OF SWAZILAND 

## SUPPLEMENTARY EXAMINATION 2012/2013

## IDE-Dip. Comm

| TITLE OF PAPER | $:$ QUANTITATIVE TECHNIQUES |
| :--- | :--- |
| COURSE NUMBER | $:$ MS 202 |
| TIME ALLOWED | $:$ THREE (3) HOURS |
| INSTRUCTIONS | $: 1$. THIS PAPER CONSISTS OF |
|  | 2. ANSWER ANY FIVE QUESTIONS. |
|  | 3. NON PROGRAMMABLE |

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

## QUESTION 1

1. (a) A firm produces two types of calculators, $x$ thousand of type $A$ and $y$ thousand of type $B$ per year. The weekly demand and cost equations are:

$$
\begin{aligned}
p & =230-9 x+y \\
q & =130+x-4 y \\
C(x, y) & =200+80 x+30 y
\end{aligned}
$$

i. Determine how many of each type of calculator should be produced per year to maximize profit?
[5 marks]
ii. What is the maximum profit?
iii. Verify that this is indeed the maximum profit.
(b) i. Find and classify all critical points of the function $f(x, y)=x^{3}+y^{3}-6 x y$.
[10 marks]

## QUESTION 2

2. (a) Solve the linear system

$$
\begin{aligned}
x+2 y+z & =0 \\
x-y-z & =-1 \\
2 x+y+z & =2
\end{aligned}
$$

using Cramer's rule.
[8 marks]
(b) An economy is based on three industries: labour, transportation and food. Each $\$ 1$ in labour requires $40 c$ in transportation and $20 c$ in food, while each $\$ 1$ in transportation takes 50 c in labour and 30 c in transportation, and each $\$ 1$ in food production uses $50 c$ in labour, $5 c$ in transportation, and $35 c$ in food.
Let the demand for the current production period be $\$ 8,000$ labour, $\$ 12,000$ transportation, and $\$ 9,000$ food.
Find the production schedule for the economy.
[12 marks]

## QUESTION 3

3. A company manufactures desks and chairs. Each desk requires 5 hours of carpentry and 1 hours of finishing. Similarly, a chair needs 1 hour of carpentry and 1 hour of finishing. During each production period, the carpentry and finishing departments can only work for up to 9 hours, 5 hours respectively. The company makes E2 profit per desk and E1 profit per chair.
(a) The problem is to determine the number of desks and chairs that should be made in order to maximize profits. Formulate this as a linear programming problem.
(b) Solve linear programming problem by the simplex method.
[12 marks]

## QUESTION 4

4. Two dietary drinks are used to supply protein and carbohydrates. The first drink provides 1 unit of protein and 3 units of carbohydrates in each litre. The second drink supplies 2 units of protein and 2 units of carbohydrates in each litre. An athlete requires 3 units of protein and 5 units of carbohydrates. The first drink costs E1 per litre and the second costs E3 per litre.
(a) The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem. . [8 marks]
(b) Solve linear programming problem by maximizing the dual. [12 marks]

## QUESTION 5

5. An wood processing company ships timber from 3 plantations, $X, Y$ and $Z$, to its 3 factories, $A, B$ and $C$. Table (1) shows the demand, availabilities and unit costs of transportation.

|  | $A$ | $B$ | $C$ | Availability |
| :---: | :---: | :---: | :---: | :---: |
| $X$ | 8 | 6 | 9 | 40 |
| $Y$ | 6 | 3 | 8 | 60 |
| $Z$ | 10 | 7 | 9 | 140 |
| Demand | 180 | 40 | 20 |  |

Table 1: Demand, supply and unit cost values
Starting with the north-west corner solution and using the stepping-stone method, determine the transportation pattern that minimises the total cost.
What is the minimum total transportation cost?

## QUESTION 6

6. (a) A company wishes to assign its employees $1,2,3,4$ to 4 different projects.

The assignment costs are given as follows:

| Cost | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 21 | 20 | 39 | 36 |
| 2 | 25 | 22 | 24 | 25 |
| 3 | 36 | 22 | 36 | 26 |
| 4 | 34 | 21 | 25 | 39 |

Determine the optimal assignment schedule that minimizes the total cost.
[10 marks]
(b) A company has 4 employees $1,2,3,4$ to assign to 4 projects $A, B, C, D$ based on the following scores:

| Score | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 90 | 65 | 95 | 40 |
| 2 | 70 | 60 | 80 | 75 |
| 3 | 85 | 40 | 80 | 60 |
| 4 | 55 | 80 | 65 | 55 |

Determine the optimal assignment schedule that maximizes the total score.
[10 marks]

## QUESTION 7

7. (a) A debt of E1200 is to be paid off by payments of E500 in 45 days, E300 in 100 days and a final payment of E436.92. Interest is at $11 \%$ and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made?
[6 marks]
(b) A T.V set can be purchased using only one of two options. The first option is to pay E1100 cash. The second option requires a down payment of $E 450$ followed by payments of $E 48$ every month for 18 months. If interest charged is at rate $5 \%$ compounded monthly, are the two options equivalent? Justify your answer.
[8 marks]
(c) How much should you deposit in an account paying $6 \%$ compounded semiannually in order to be able to withdraw $E 1000$ every 6 months for the next 3 years?
[6 marks]
