University of Swaziland

Final Examination – May 2014

BA in Social Science I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MS012

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Evaluate

    i. $\lim\limits_{x \to -4} \left(2x^2 - 3x - 1\right)$ [2 marks]

    ii. $\lim\limits_{x \to -2} \left(\dfrac{x^2 - 4}{x^2 + 4}\right)$ [2 marks]

    iii. $\lim\limits_{x \to 5} \left(\dfrac{x - 5}{5x - x^2}\right)$ [3 marks]

    iv. $\lim\limits_{x \to \infty} \left(\dfrac{2x^2 - x + 3}{5 + 2x - x^2}\right)$ [4 marks]

A.2 a. State the *limit definition* of the derivative of the function $f(x)$. [2 marks]

    b. Use the limit definition to find $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ given

$$f(x) = 3x^2 - 7.$$ [7 marks]

    c. Find y' if

        i. $y = x + 5x^6$ [2 marks]

        ii. $y = 12X^{\frac{2}{3}} - 25X^{-\frac{3}{5}}$ [3 marks]

        iii. $y = 3 - \dfrac{3}{x^2}$ [3 marks]

        iv. $y = e^{3x+2}$ [2 marks]

        v. $y = \ln(5x)$ [3 marks]

A.3 a. State the *Fundamental Theorem of Calculus*. [3 marks]

    b. Integrate

        i. $\displaystyle\int_{1}^{4} \left(3 - 4x + 3x^2\right)\mathrm{d}x$ [5 marks]

        ii. $\displaystyle\int \left(15X^{\frac{2}{3}} - \dfrac{3}{X}\right)\mathrm{d}X$ [3 marks]

        iii. $\displaystyle\int \left(\dfrac{2}{x^2} - \dfrac{4}{x^5}\right)\mathrm{d}x$ [3 marks]

        iv. $\displaystyle\int e^{0.5x}\mathrm{d}x$ [3 marks]

Section B

Answer ANY 2 Questions in this section

B.4 a. Find the value of the limit
$$\lim_{x \to 2} \frac{x^3 - 8}{3x^2 - 4x - 4}$$
[7 marks]

b. Find the indicated derivative

i. $y = \left(8x^2 - 3\right)^9$ $\qquad$ y' $\qquad$ [3 marks]

ii. $y = (3x - 1)e^{-4x}$, $\qquad$ y' $\qquad$ [4 marks]

iii. $y = \dfrac{2 + 3x}{4 - 7x}$, $\qquad$ y' $\qquad$ [6 marks]

iv. $y = 16\sqrt{x} - \dfrac{1}{x}$, $\qquad$ y''' $\qquad$ [5 marks]

B.5 a. Consider the function
$$y = 4 + 27x - x^3.$$

i. Find the *stationary points* and classify them as relative maxima or minima. [10 marks]

ii. Find the y-intercept. [2 marks]

iii. Make a sketch of the graph of y. [4 marks]

b. The profit (in Emalangeni) of a company is given by
$$P(x) = 75x - 0.015x^2 - 10000$$

where x is the number of units sold per month. Find the

i. profit if the number of units sold is 1,500 [3 marks]

ii. the maximum monthly profit. [6 marks]

B.6 a. By first making the substitution $u = x^2 + 4$, evaluate the integral

$$\int 20x\left(x^2 + 4\right)^9 dx.$$ [10 marks]

b. Use the method of partial fractions to integrate

$$\int \frac{x + 4}{(x - 2)(x + 1)} dx.$$ [15 marks]

B.7 a. Use the method of tabular integration to evaluate

$$\int 81x^2 \cos 3x \, dx.$$ [12 marks]

b. Find the area of the shaded region in the figure blow. [13 marks]

$y = x^2 - 7x$

$y = 9x - x^2$