13

University of Swaziland

Final Examination, December 2013

B.A.S.S. I , B.Comm I, D.Comm I (IDE)

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Code : MS101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

1.1. State the remainder theorem. [2]

1.2. By using the factor theorem, state which of the following

(a) $x + 2$. [1]

(b) $2x - 3$. [1]

are factors of the polynomial

$$P(x) = 2x^3 - 3x^2 - 2x + 3.$$

1.3. Use the long division method to find the quotient and remainder when

$$P(x) = x^4 + 2x^2 + 5x + 1$$

is divided by $D(x) = x - 1$. [5]

1.4. Solve the following equations

(a) $\log_2(x + 1) - \log_2(2 - x) = 3$. [3]

(b) $2^{x+1} = 3^{x-1}$. [3]

(c) $x + \sqrt{-\frac{1}{25}} = 0$. [3]

1.5. Given

$$A = \begin{bmatrix} -1 & 3 & -4 \\ 0 & 2 & 1 \\ 2 & -3 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$$

calculate

(a) $|A|$. [3]

(b) $A^T B$. [4]

1.6. Find the equation of a straight line passing through $(1, -1)$ and is perpendicular to the line $3y - 6x = 1$. [5]

1.7. Given $z_1 = 4 + 2i$, $z_2 = 5 + i$ and $z_3 = -8 + 3i$. Write

$$\frac{z_1}{z_2} + \bar{z}_3$$

in the form $a + ib$. [6]

1.8. If $\tan\theta = \frac{3}{4}$, find all possible values of $\sin\theta$ and $\cos\theta$. [4]

SECTION B: ANSWER ANY 3 QUESTIONS

2. Given the following polynomial

$$P(x) = x^4 - x^3 - 19x^2 + 49x - 30$$

(a) List all the possible roots of $P(x)$. [3]

(b) Find the number of positive real zeros(roots) of $P(x)$. [3]

(c) Find the number of negative real zeros(roots) of $P(x)$. [3]

(d) Use the factor theorem and synthetic division (ONLY) to find the factors of $P(x)$. [11]

3. (a) Given an arithmetic progression

$$50, 46, 42, \cdots, -62$$

find the number of terms that are in the progression. [5]

(b) John wants to buy a new car after 5 years that will cost him $E100000$. If he only has $E60000$ to deposit now, what interest rate is required for it to increase to $E100000$ in 5 years if the interest is compounded monthly. [5]

(c) Without using a calculator find the exact value of $\sin 1485°$. [5]

(d) Find the centre and radius of a circle defined by

$$x^2 - 6x + y^2 + 4y - 3 = 0.$$

[5]

4. (a) Given the following expansion

$$\left(2x^3 + \frac{1}{2x^2}\right)^{20},$$

find the

i. first 3 terms. [5]

ii. term involving x^{-5}. [3]

iii. constant term. [3]

(b) Use Cramer's rule to solve the following system of equations

$$\begin{array}{rcrcrcl} 2x & - & y & - & 2z & = & -3 \\ x & + & 3y & + & z & = & -1 \\ 5x & - & 4y & + & 3z & = & 10. \end{array}$$

[9]

5. (a) Prove the following trigonometric identity

$$\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta.$$

[5]

(b) Solve the following equations

i. $\sin x - 2\sin x \cos x = 0, \qquad 0° \leq x \leq 360°$ [7]

ii. $\log_x\left(\frac{1}{8}\right) = 3.$ [4]

(c) Convert $1.701010101\cdots$ into an equivalent fraction. [4]

6. (a) Prove by mathematical induction that the formula

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$$

is valid for all positive integers. [10]

(b) Give the binomial expansion for $\frac{1}{\sqrt[5]{1-2x}}$ up to and including x^3 (where x is small). Use this expression to find $\frac{1}{\sqrt[5]{0.93}}$. (6 decimal places) . [10]

END