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University of Swaziland

Supplementary Examination, July 2014

B.A.S.S. I , B.Comm I, D.Comm I (IDE)

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Code : MS101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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SECTION A: ANSWER ALL QUESTIONS

1.1. State the factor theorem. [2]

1.2. By using the remainder theorem state which of the following values

(a) $x = -\frac{5}{4}$. [1]

(b) $x = \frac{7}{2}$. [1]

are roots of the polynomial

$$P(x) = 6x^4 - 35x^3 + 27x^2 + 107x - 105.$$

1.3. Use the long division method to find the quotient and remainder when

$$P(x) = 14x^4 - 16x^3 - 54x^2 - 8$$

is divided by $D(x) = x - 3$. [5]

1.4. Solve the following equations

(a) $x^{\frac{2}{3}} = 8$. [3]

(b) $\log_2(2 + x) - \log_2(2x - 1) = 2$. [4]

(c) $3^{1-2x} = 5^{2-x}$. [3]

(d) $x^2 + 3x + 9 = 0$. [3]

1.5. Without using a calculator, find the exact value of $\sin 1215°$. [3]

1.6. Find three numbers in an arithmetic progression such that their sum is 27 and their product is 504. [5]

1.7. Find the determinant of the matrix

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 1 \\ 4 & -3 & 2 \end{pmatrix}.$$

[5]

1.8. Given $z_1 = 2 + 3i$, $z_2 = 1 - i$ and $z_3 = 3 + 4i$, express $\dfrac{\bar{z}_1 + z_2}{z_3}$ in the form $a + ib$. [5]

SECTION B: ANSWER ANY 3 QUESTIONS

2. Given the following polynomial

$$P(x) = 3x^4 - x^3 - 29x^2 + 9x + 18$$

(a) List all the possible roots of $P(x)$. [3]

(b) Find the number of positive real zeros(roots) of $P(x)$. [3]

(c) Find the number of negative real zeros(roots) of $P(x)$. [3]

(d) Use the Remainder theorem and synthetic division (ONLY) to find the roots of $P(x)$. [11]

3. (a) Prove the following trigonometric identity

$$\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta.$$

[5]

(b) Solve the following equations

i. $2\cos^2 x + 3\sin x - 3 = 0, \qquad 0° \leq x \leq 360°$ [7]

ii. $z^2 - (3 - i)z + 4 = 0.$ [8]

4. (a) Use Cramer's rule to solve the following system of equations

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & 1 \\ x & - & y & - & z & = & 0 \\ 2x & + & y & + & z & = & 3. \end{array}$$

[10]

(b) Find the sum of the following series

$$1 + 5 + 9 + ... + 793.$$

[5]

(c) Convert $2.7343434343434\cdots$ into and equivalent fraction. [5]

5. (a) Given the following expansion

$$\left(\frac{1}{2x} + x^2\right)^{15},.$$

Find the

i. sixth term [3]

ii. constant term [4]

iii. term involving x^{15}. [4]

(b) Find the equation of a straight line passing through the intersection of $3x - y = 9$ and $x + 2y = -4$, perpendicular to $3 = 4y + 8x$. [9]

6. (a) Find the centre and radius of a circle defined by the equation

$$x^2 - 6x + y^2 + 10y + 25 = 0.$$

[5]

(b) Give the binomial expansion for $\sqrt[4]{1+5x}$ up to and including x^2 (where x is small). Use this expression to find $\sqrt[4]{1.05}$. (6 decimal places) [5]

(c) Prove by mathematical induction that the formula

$$4 + 8 + 12 + \cdot + 4n = 2n(n+1)$$

is valid for all positive integers. [10]

END