University of Swaziland

Final Examination, April 2014

B.A.S.S. I , B.Comm I, D.Comm I (IDE)

Title of Paper : Calculus for Business and Social Sciences

Course Code : MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: COMPULSORY QUESTIONS

QUESTION 1

(a) Evaluate the following limits

    (i) $\lim\limits_{x \to -5} \dfrac{x^2 + 3x - 10}{x - 5}$.   [3 marks]

    (ii) $\lim\limits_{x \to \infty} \dfrac{x^2 + 6x - 1}{x^2 - 16}$.   [3 marks]

(b) Use the limit definition of the derivative to find the derivative $f'(x)$ of the function $f(x) = 4x^2$ [5 marks]

(c) If $\lim\limits_{x \to 0} f(x) = 2$, then $f(0) = 2$. Determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

                                            [3 marks]

(d) Find the derivatives of the following functions

    (i) $f(x) = \dfrac{x + 1}{(x + 2)^2}$.   [3 marks]

    (ii) $y = e^{2x+5}$.   [3 marks]

(e) Find the slope and equation of the tangent line to the graph of $f(x) = x + \dfrac{1}{x}$ at the point $(1, 2)$.   [5 marks]

QUESTION 2

(a) Evaluate the following integrals

    (i) $\displaystyle\int_0^1 (x^2 + 1)^2 \, dx$.   [5 marks]

    (ii) $\displaystyle\int \sin x \, e^{\cos x} \, dx$.   [5 marks]

(b) Find the area of the region bounded by $f(x) = 4 - x^2$ and the x-axis.   [5 marks]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 3

(a) Evaluate the following limits

    (i) $\lim\limits_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16}$.   [6 marks]

    (ii) $\lim\limits_{x \to \infty} \sqrt{4x^2 - 2x + 1} - 2x$.   [6 marks]

(b) Show that the function
$$f(x) = \begin{cases} 1, & x = 0; \\ \frac{\sin x}{x}, & x \neq 0. \end{cases}$$

is continuous at $x = 0$ [8 marks]

QUESTION 4

Find the derivative of the following functions

(a) $f(x) = e^{3x} \sin(4x^2 + 10x - 3)$.   [5 marks]

(b) $f(x) = 3^{x \cos x}$.   [5 marks]

(c) $y = (x^4 + 3x^2)^{10}$.   [5 marks]

(d) $y = \ln\left(\dfrac{\sin x}{e^x}\right)$.   [5 marks]

QUESTION 5

For the curve given by
$$f(x) = x^3 - 12x + 12$$
find the point where the curve assumes a

    (a) local minimum   [2 marks]

    (b) local maximum   [2 marks]

    (c) a point of inflection   [2 marks]

Find the intervals where the curve is

    (d) increasing   [2 marks]

    (e) decreasing   [2 marks]

    (f) concave upward   [3 marks]

    (g) concave downward   [3 marks]

    (h) using the above information (a) to (g), sketch the curve f(x)   [4 marks]

QUESTION 6

The weekly demand for Pulsar DVD recorders is given by the demand equation
$$p = -0.02x + 300 \quad \text{for} \quad 0 \le x \le 15,000$$
where p denotes the wholesale unit price in Emalangeni and x denotes the quantity demanded. The weekly total cost function associated with manufacturing these recorders is
$$C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70,000 \text{ Emalangeni.}$$

    (a) Find the revenue function and the profit function.   [3 marks]

    (b) Find the marginal cost function, the marginal revenue function, and the marginal profit function.   [6 marks]

    (c) Find the marginal average cost function.   [2 marks]

Compute

    (d) marginal cost when $x = 3000$ and interpret your results   [3 marks]

    (e) marginal revenue when $x = 3000$ and interpret your results   [3 marks]

    (f) marginal profit when $x = 3000$ and interpret your results   [3 marks]

QUESTION 7

(a) Evaluate the following integrals

   (i) $\displaystyle\int_0^1 x \cos x \, dx$ [5 marks]

   (ii) $\displaystyle\int \frac{x}{x^2 - 4x + 3} \, dx$ [5 marks]

(b) Given the demand function
$$D(x) = 20 - 0.05x$$
and the supply function
$$S(x) = 2 + 0.0002x^2$$
find

   (i) the equilibrium price   [3 marks]

   (ii) the consumer's surplus   [3 marks]

   (iii) the producer's surplus.   [4 marks]

END OF EXAMINATION PAPER