

University of Swaziland

Supplementary Examination, July 2014

B.A.S.S. I , B.Comm I, D.Comm I (IDE)

Title of Paper : Calculus for Business and Social Science

Course Code : MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
 - a. **SECTION A(COMPULSORY): 40 MARKS**
Answer ALL QUESTIONS.
 - b. **SECTION B: 60 MARKS**
Answer ANY THREE questions.
Submit solutions to **ONLY THREE** questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

(a) Evaluate the following limits

(i) $\lim_{x \rightarrow 0} \frac{x^2 - x}{x}$. [3 marks]

(ii) $\lim_{x \rightarrow \infty} \frac{4x^2 - x + 15}{3x^2 + 7x - 15}$. [3 marks]

(b) Use the limit definition of the derivative to find the derivative $f'(x)$ of the function

$$f(x) = \sqrt{x}.$$

[5 marks]

(c) Find a value for k which will make the function

$$f(x) = \begin{cases} x^3 - 1, & x < 2; \\ kx^2, & x \geq 2. \end{cases}$$

continuous at $x = 2$.

[3 marks]

(d) Find the derivatives of the following functions

(i) $f(x) = e^{4x} \sin(2x)$. [3 marks]

(ii) $f(x) = \frac{\ln(x+1)}{x}$. [3 marks]

(e) Find the slope and equation of the tangent line to the graph of

$$f(x) = (2x + 1)^2$$

at $x = 1$.

[5 marks]

QUESTION 2

(a) Evaluate the following integrals

(i) $\int \left(x^3 + \frac{3}{x} + \cos(2x) \right) dx$. [5 marks]

(ii) $\int \frac{3x^2}{x^3 + 1} dx$. [5 marks]

(b) Find the area of the region bounded by

$$f(x) = -x^2 + x + 6$$

and the x -axis.

[5 marks]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 3

(a) Evaluate the following limits

(i) $\lim_{x \rightarrow 3} \frac{3-x}{\sqrt{x}-\sqrt{3}}$ [6 marks]

(ii) $\lim_{x \rightarrow 2} \frac{x^3 + 4x - 2x^2 - 8}{x-2}$ [7 marks]

(b) State the three conditions which guarantee continuity of a function $f(x)$ at the point $x = c$. Using these properties test whether the function

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1; \\ 2, & x = 1 \end{cases}$$

is continuous at the point $x = 1$. [7 marks]

QUESTION 4

Find the derivative of the following functions

(a) $f(x) = x^x$. [5 marks]

(b) $f(x) = \left(\frac{x+1}{x}\right)^2$. [5 marks]

(c) $f(x) = \ln(2x^3 - 3)$. [5 marks]

(d) $f(x) = e^{x+1} \cos(x^3 - 1)$. [5 marks]

QUESTION 5

(a) Given the function

$$y = 2x^3 - 15x^2 + 36x - 10,$$

find the

(i) local maximum. [2 marks]

(ii) local minimum. [2 marks]

(iii) point of inflection. [2 marks]

(b) Find the intervals where the curve is

(i) increasing. [2 marks]

(ii) decreasing. [2 marks]

(iii) concave up. [3 marks]

- (iv) concave down. [3 marks]
- (c) Use all the information obtained in (a) and (b) to sketch the graph of the function. [4 marks]

QUESTION 6

- (a) Given the demand function $D(x)$ and the supply function $S(x)$

$$p = D(x) = 300 - 6x - x^2, \quad p = S(x) = x^2 + 4x$$

find the

- (i) equilibrium price [2 marks]
- (ii) consumer surplus [5 marks]
- (ii) producer surplus [5 marks]
- (b) A cattle owner has 800 meters of fence which he wishes to use to make a rectangular holding region in which cattle will graze. If the region will border an existing fence, calculate the largest area that can be enclosed and find the dimensions of the region. [8 marks]

QUESTION 7

- (a) Evaluate the following integrals

(i) $\int x \ln x dx.$ [5 marks]

(ii) $\int_0^5 \frac{2}{x^2 + 5x + 6} dx.$ [5 marks]

- (b) A company manufactures x carts per month. If the monthly cost and price-demand functions are given by

$$C(x) = \frac{x^2}{100} + \frac{x}{2} + 8, \quad p(x) = -\frac{x}{200} + 1.$$

Find the following, for each month.

- (i) Revenue function. [2 marks]
- (ii) Find the marginal cost when $x = 30$. Interpret your results. [4 marks]
- (iii) Find the marginal profit when $x = 30$. Interpret your results. [4 marks]

END