| UNIVERSITY OF SWAZILAND <br> MAIN EXAMINATION 2013/2014 |
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| BComm. /BEd. |
| TITLE OF PAPER : Quantitative Techniques |
| COURSE NUMBER : MS 202 |
| TIME ALLOWED $: 3$ HOURS |
| SPECIAL REQUIREMENTS : NONE. NOT EVEN GRAPH IS REQUIRED. |
| Instructions <br> (a) Candidates may attempt: <br> (i) ALL questions in Section A and <br> (ii) At most THREE questions in Section B. <br> (b) Each question should start on a fresh page. <br> (c) THERE ARE NO SPECIAL REQUIREMENTS FOR THIS PAPER. |
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## SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. A shop sells $x \mathrm{~kg}$ of sugar and $y \mathrm{~kg}$ of salt in a month at prices $p$ Emalangeni per kg and $q$ Emalangeni per kg respectively. The demand equations for the company are

$$
x=300+2 p+q, \quad y=200+p+q
$$

and its cost function is $1000-3 x y$.
(a) Determine the monthly profit function $P(x, y)$ for the company.
(b) Evaluate $P_{\mathbf{x}}(40,10)$ and interpret your results.

A2. Evaluate the determinant $\left|\begin{array}{ccc}2 & 8 & 5 \\ 4 & 12 & 8 \\ 3 & 4 & 4\end{array}\right|$ using elementary row operations.

A3. Two dietary drinks are used to supply protein and carbohydrates. The first drink provides 2 units of protein and 3 units of carbohydrates in each litre. The second drink supplies 5 units of protein and 7 units of carbohydrates in each litre. An athlete requires 12 units of protein and 17 units of carbohydrates. The first drink costs E21 per litre and the second costs E50 per litre.

The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem.

A4. A company wishes to assign its employees $A, B, C, D$ to 4 different training courses $1,2,3,4$ based on their skills. The assignment costs are given as follows:

| Cost | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| A | 10 | 8 | 4 | 6 |
| B | 6 | 4 | 12 | 8 |
| C | 14 | 10 | 8 | 2 |
| D | 4 | 14 | 10 | 8 |

Determine the optimal assignment schedule that minimizes the total cost.

A5. A man wishes to set up a fund to provide $E 50,000$ for the purchase of a second hand truck at the end of 5 years when he retires. If equal deposits are made at the end of every 6 months in a fund earning $7 \%$ interest converted semi-annually, determine the value of each deposit.

## SECTION B (60 marks)

Candidates may attempt THREE questions being careful to number them B6 to B10.

B6. (a) Find and classify all stationary points of the function $12 x+12 y-4 x^{3}-4 y^{3}$. [10]
(b) A company manufactures $x$ stools and $y$ tables per week at a cost of $C(x, y)=8 x^{2}-x y+12 y^{2}$. Due to shipping considerations, it is necessary that $x+y=42$. Use the Method of Lagrange Multipliers to determine values of $x$ and $y$ which minimize $C$. Verify your results.

B7. (a) Solve the linear system

$$
\begin{aligned}
2 x_{1}+6 x_{2}+4 x_{3} & =-2 \\
4 x_{2}+x_{3} & =2 \\
x_{1}-2 x_{2} & =4
\end{aligned}
$$

using Gaussian elimination.
(b) A company has 3 interacting brances $X, Y$ and $Z$. Branch $X$ consumes E 0.15 of own output, E 0.25 of $Y$ output and E0.1 of $Z$ output for every E1 it produces. Branch $Y$ consumes E0.25 of $X$ output, E0.25 of own output and E0.5 of $Z$ output per E1 of output. Branch $Z$ consumes E 0.1 of $X$ output, E0. 3 of $Y$ output and E0.1 of own output per E1 of output. How much should each branch produce per month in order to meet exactly a monthly external demand of E80000 for $X$ product, E50 000 for $Y$ product and E65000 for $Z$ product?

B8. A company manufactures chairs and desks. A chair requires 2 hours of carpentry, 1 hour of painting and 1 hour of finishing. A desk requires 1 hour of carpentry, 2 hours of painting and 3 hours of finishing. During each production period, the carpentry, painting and finishing departments can only work for up to 20 hours, 12 hours and 15 hours respectively. Profit on a chair is E15 and that on a gesk is E36.
(a) The problem is to determine the number of chairs and desks that should be made in order to maximize profits. Formulate this as a linear programming problem. [8]
(b) Solve the resultant linear programming problem using the graphical method. [12] THE USE OF GRAPH PAPER HERE IS STRICTLY PROHIBITED AND IT MAY RESULT IN LOSS OF MARKS.

B9. (a) A loan of $E 2300$ is due in 8 months with interest charged at $4.5 \%$ per annum. The debtor makes a first payment of E1100 in 3 months, followed by a payment of E690 in 6 months. Find the balance payable on due date under the Merchant's rule.
(b) A T.V set can be purchased using only one of two options. The first option is to pay $E 4500$ cash. The second option requires a down payment of $E 1500$ followed by payments of $E 800$ every month for 6 months. If interest charged is at rate $5 \%$ compounded monthly, are the two options equivalent?
[8]
(c) What sum of money should be set aside to provide an income of $E 800$ every three months for the next 3 years if the money earns interest at rate $7 \%$ compounded quarterly?

B10. A car dealership has a contract to move 384 vehicles from 3 manufacturing plants $A, B, C$ to 3 of its showrooms $D, E, F$ for selling. The cost of moving 1 vehicle from $A$ to each of $D, E, F$ is $\mathrm{E} 3, \mathrm{E} 3, \mathrm{E} 2$ respectively. Similarly, the respective costs of moving 1 vehicle from $B$ are $\mathrm{E} 4, \mathrm{E} 2, \mathrm{E} 3$, and those from $C$ are $\mathrm{E} 3, \mathrm{E} 4, \mathrm{E} 3$. Vehicles available for distribution from $A, B, C$ are $100,160,124$ respectively. Vehicles required at $D, E, F$ are $120,120,144$ respectively.
Determine the optimum distribution strategy for this company to minimize total transportation cost. Evaluate the minimum total transportation cost.

