University of Swaziland

Final Examination – November 2014

BASS I

Title of Paper : Elementary Quantitative Techniques I
Course Number : MS011
Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 (out of 4) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. Simplify

i. $\dfrac{2}{x+2} - \dfrac{1}{x+1}$, [5 marks]

ii. $\dfrac{2x^2 - x}{2x^2 + 3x - 2}$, [5 marks]

iii. $\dfrac{10x^4b^{-3}}{3a^{-2}y^{-5}} \div \dfrac{25x^{-3}b^2}{27a^4y^{-3}}$ (expressing your answer in terms of positive indices), [7 marks]

b. Use a calculator to compute (for non-exact answers, express correct to 3 s.f.)

i. ${}_{24}C_{13}$, [1 marks]

ii. $\log 4\,500$, [1 marks]

iii. $\ln 5\,000$, [1 marks]

iv. $\log_3 500$. [3 marks]

c. Use the *quadratic formula* to solve (expressing your answer correct to 3 s.f.)

$$2x^2 + 25x = 9.$$ [5 marks]

d. Solve the simultaneous equations

$$\begin{aligned} 2x + 5y &= 31, \\ 3x - 2y &= -20. \end{aligned}$$ [7 marks]

e. Given that

$$A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 2 & 3 \end{pmatrix},$$

work out

i. $2A + 3B^T$, [5 marks]

ii. $A^T B$, [5 marks]

iii. $C^T A$. [5 marks]

Section B

Answer ANY 2 Questions in this section

B.1 Use Cramer's rule to solve

$$\begin{aligned} 2x - y + z &= 8 \\ x + y + z &= 4 \\ 2y - z &= -5. \end{aligned}$$

[25 marks]

B.2 a. For the *arithmetic progression*

$$10,\ 14,\ 18\cdots,$$

find

i. the formula for the n-th term, [2 marks]

ii. the 50th term using the formula in i., [2 marks]

iii. the sum of the first 25 terms. [4 marks]

b. Find the value of

i. $10 + 20 + 30 + \cdots + 5,000$ [4 marks]

ii. $\sum_{n=1}^{50}(2n+3)$ [6 marks]

c. Use *synthetic division* to work out

$$\frac{x^3 - 2x^2 + 3x - 7}{x - 2}.$$ [4 marks]

d. Given that $x + 2$ is a factor of $P(x) = x^3 + Ax^2 + 2x - 4$, find the value of A. [3 marks]

B.3

a. Consider the straight line between $A(2,-3)$ and $B(-2,5)$. Find

i. the length of AB correct to 3 s.f., [2 marks]

ii. the gradient of AB, [2 marks]

iii. the equation of AB, [4 marks]

iv. the equation of the perpendicular bisector of AB. [4 marks]

b. Use the ***binomial theorem*** to expand and simplify term by term

$$(x+3)^4.$$ [6 marks]

c. In the binomial expansion of

$$(x+y)^{20},$$

find

i. the first 4 terms, [4 marks]

ii. the 16th term. [3 marks]

B.4

a. Express

i. $4^3 = 64$ in terms of logs, [2 marks]

ii. $\log_3 81 = 4$ in exponential form. [2 marks]

b. Solve (for non-exact answers, express correct to 3 s.f.)

i. $5^{x-1} = 625$, [3 marks]

ii. $3^x = 500$, [4 marks]

iii. $\log_7(2x-1) = 2$, [4 marks]

c. A new car valued at E430,000 is bought on 01 January 2014. Its value subsequently depreciates according to the formula

$$V(t) = 430,000e^{-0.085t},$$

where t is the age of the car in years. Find the

i. value of the car on 31 December 2015, [4 marks]

ii. the date when the car reaches its ***half-life***. [6 marks]