# University of Swaziland

## Final Examination, December 2014

## B.A.S.S. I , B.Comm I, D.Comm I (IDE), B. Ed

**Title of Paper** : Algebra, Trigonometry and Analytic Geometry

**Course Code** : MS101

**Time Allowed** : Three (3) Hours

**Instructions**

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      **Submit solutions to ONLY THREE questions in Section B.**
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

# SECTION A: ANSWER ALL QUESTIONS

## QUESTION 1

**a.** State the remainder theorem. [2]

**b.** Using the remainder theorem find the remainder when the polynomial $P(x) = 3x^4 + x^3 - 4x^2 + 5$ divided by $x - 1$. [2]

**c.** Using the long division method find the quotient and remainder when

$$P(x) = x^4 + 3x^3 - 2x + 4$$

is divided by $D(x) = x + 1$. [4]

**d.** The polynomial $P(x) = x^3 + Ax^2 + Bx + 6$ has $(x - 2)$ and $(x + 1)$ as factors. Find the values of A and B. [4]

**e.** Solve the following equations (without using a calculator)

**i.** $\log_3(x + 1) - \log_3(x - 1) = 1$. [3]

**ii.** $4^{2x} = 5^{x+1}$. [3]

**iii.** $x + \left(\frac{8}{27}\right)^{-\frac{1}{3}} = 0$. [3]

**f.** Calculate $(A - B)C^T$ if the matrices $A, B$ and $C$ are given by

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 4 \\ 6 & 3 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 \\ -2 & 4 \\ 1 & 3 \\ 3 & -1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & 0 \\ 3 & 2 \end{bmatrix}$$

[5]

**g.** Find the equation of a straight line passing through $(-1, 1)$ and is parallel to the line $2x + y - 1 = 0$. [4]

**h.** If $\theta$ is an acute angle and $\sin\theta = \frac{12}{13}$, find all possible values of $\cos\theta$ and $\tan\theta$. [5]

**i.** Evaluate $\frac{(1 + i)(2 + 3i)}{1 - i}$ and write the solution in the form $a + bi$. [5]

# SECTION B: ANSWER ANY 3 QUESTIONS

## QUESTION 2

Given the following polynomial

$$P(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$$

**i.** List all the possible roots of $P(x)$. [3]

**ii.** Find the number of positive real zeros(roots) of $P(x)$. [3]

**iii.** Find the number of negative real zeros(roots) of $P(x)$. [3]

**iv.** Use the factor theorem and synthetic division (ONLY) to find the factors of $P(x)$.[11]

## QUESTION 3

**i.** A new car costs $E100,000$. Assume that it depreciates 24% the first year, 20% the second year, 16% the third year, and continues in the same manner for 6 years. If all depreciations apply to the original cost, what is the value of the car in 6 years? [5]

**ii.** How long will it take for money in an account that is compounded continuously at 8% interest to double? [5]

**iii.** The fourth term of a geometric series is 16 and the second term is 2. Find the first term and a common ratio? [5]

**iv.** Express $\log_b 2x + 3(\log_b x - \log_b y)$ as a single logarithm with a coefficient of 1. [5]

## QUESTION 4

**i.** Find the $6^{th}$ term in the expansion of

$$(3a^2 + 2b)^{10}.$$

[5]

**ii.** Write the first four terms in the expansion of $(1 + x)^{-\frac{1}{3}}$. [5]

iii Use Cramer's rule to solve the following system of equations

$$\begin{array}{rcrcrcl} 3x & + & 2y & + & z & = & 10 \\ 2x & + & 3y & - & z & = & 5 \\ x & + & y & + & 3z & = & 12. \end{array}$$

[10]

## QUESTION 5

**i.** Find the value of $\sqrt{120}$ correct to four significant figures (use binomial expansion). [7]

**ii.** Prove the following trigonometric identity

$$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta.$$

[7]

**iii.** Convert 2.071613613613613..... into and equivalent fraction. . [6]

## QUESTION 6

**i.** Prove by mathematical induction that the formula

$$5 \cdot 6 + 5 \cdot 6^2 + 5 \cdot 6^3 + \ldots + 5 \cdot 6^n = 6 \cdot (6^n - 1)$$

is valid for all positive integers. [10]

**ii.** Given the points $A = (4, 0)$ and $B = (6, 4)$. Find the equation of a circle with centre A and passing through the point B. [5]

**iii.** Find the equation of a straight line passing through the intersection of $3x - y = 9$ and $x + 2y = -4$, perpendicular to $3 = 4y + 8x$. [5]

**END**