## University of Swaziland

## Supplementary Examination, July 2015

## B.A.S.S. I , B.Comm I, D.Comm I (IDE), B. Ed

| Title of Paper | $:$ Algebra, Trigonometry and Analytic Geometry |
| :--- | :--- |
| Course Code | $:$ MS101 |
| Time Allowed | $:$ Three (3) Hours |

## Instructions

1. This paper consists of TWO sections.
a. SECTION A(COMPULSORY): 40 MARKS

Answer ALL QUESTIONS.
b. SECTION B: 60 MARKS

Answer ANY THREE questions.
Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth $20 \%$.
3. Show all your working.
4. Special requirements: None

This paper should not be opened until permission has been given by the invigilator.

BComm. /BEd.

TITLE OF PAPER : Quantitative Techniques
COURSE NUMBER : MS 202
TIME ALLOWED : 3 HOURS
SPECIAL REQUIREMENTS : NONE; NOT EVEN GRAPH PAPER.

## Instructions

(a) Candidates may attempt:
(i) ALL questions in Section A and
(ii) At most THREE questions in Section B.
(b) Each question should start on a fresh page.

## SECTION A: ANSWER ALL QUESTIONS

## QUESTION 1

a. State the factor theorem.
b. Use the factor theorem to show that $x+1$ is a factor of $P(x)=5 x^{5}-2 x^{4}+5 x^{3}+50 x^{2}-20 x-40$.
c. Use long division to find the quotient and remainder when

$$
\begin{equation*}
P(x)=x^{4}-x^{2}+4 x+1 \tag{4}
\end{equation*}
$$

is divided by $D(x)=x^{2}+1$.
d. Find a polynomial $P(x)$ of degree 3 with roots $-1,2,-2$ and satisfies the condition $P(1)=-6$.
e. Solve the following equations (without using a calculator)
i. $\log _{2} x=1-\log _{2}(x+1)$.
ii. $32^{x}=\left(\frac{1}{4}\right)^{x+1}$.
iii. $x+\sqrt[7]{-\frac{1}{128}}=0$.
f. Calculate $A^{T} B$ if the matrices $A$ and $B$ are given by

$$
A=\left[\begin{array}{cc}
1 & 3 \\
-2 & 1 \\
1 & -1 \\
0 & 1
\end{array}\right] \quad \text { and } \quad B=\left[\begin{array}{ll}
0 & 2 \\
1 & 2 \\
3 & 0 \\
2 & 1
\end{array}\right]
$$

g. Find the equation of a straight line passing through $(1,0)$ and is perpendicular to the line $2 y-3 x+2=0$.
h. Without using a calculator, find the exact value of $\cos 1225^{\circ}$.
i. Write the complex number $2\left(\cos 120^{\circ}+i \sin 120^{\circ}\right)$ in cartesian form $x+i y$.

## SECTION B: ANSWER ANY 3 QUESTIONS

## QUESTION 2

Given the following polynomial

$$
P(x)=6 x^{4}+x^{3}-16 x^{2}+11 x-2
$$

i. List all the possible roots of $P(x)$.
ii. Find the number of positive real zeros(roots) of $P(x)$.
iii. Find the number of negative real zeros(roots) of $P(x)$.
iv. Use the factor theorem and synthetic division (ONLY) to find the factors of $P(x)$.[11]

## QUESTION 3

i. Find the term that involves $x^{5}$ in the expansion

$$
\left(x+\frac{1}{2 x}\right)^{7}
$$

ii. Write the first four terms in the expansion of $\frac{1}{\sqrt{1-x}}$.
iii. Use Cramer's rule to solve the following system of equations

$$
\begin{array}{r}
x+2 y+z=1 \\
x-y-z=0 \\
2 x+y+z=3 .
\end{array}
$$

## QUESTION 4

i. An auditorium has 40 rows with 30 seats in the first row, 33 in the second row, 36 in the third row, and so forth. How many seats are in the auditorium?
ii. How much money will grow to $E 12000$ in 5 years at $10 \%$ if the interest is compounded monthly?
iii. Find the centre and radius of a circle defined by

$$
x^{2}+4 x+y^{2}-6 y-3=0 .
$$

iv. Solve

$$
\log _{4}\left(\log _{3}\left(x^{2}+2\right)\right)=0
$$

## QUESTION 5

i. Prove the following trigonometric identity

$$
\left(1-\cos ^{2} \theta\right)\left(1+\cot ^{2} \theta\right)=1
$$

ii. The $n^{\text {th }}$ term of 4,8 , and 16 is 1024 . Find $n$.
iii. Without using a calculator find the exact value of $\sin 1215^{\circ}$.

## QUESTION 6

i. Prove by mathematical induction that the formula

$$
3+3^{2}+3^{3}+\cdots+3^{n}=\frac{3\left(3^{n}-1\right)}{2}
$$

is valid for all positive integers.
ii. Give the binomial expansion of $(1+x)^{\frac{1}{4}}$ up to and including the term in $x^{2}$ (for small $x)$.
Use this expression to show that $\sqrt[4]{17} \approx \frac{8317}{4096}$.
END

